PRACTICAL PHYSICS

A LABORATORY MANUAL FOR

COLLEGES AND TECHNICAL SCHOOLS

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PRACTICAL PHYSICS

A LABORATORY MANUAL FOR COLLEGES AND TECHNICAL SCHOOLS

BY

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VOLUME I

PRECISE MEASUREMENTS

MEASUREMENTS IN MECHANICS AND HEAT

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are due to Professor H. C. Jones for the use of three cuts from his book on the Elements of Physical Chemistry, to Professor Wilbur M. Stein for the use of three cuts from his book on Photometric Measurements, and to Leeds & Northrup for the use of twelve cuts illustrating some of their electrical measuring instruments.

THE AUTHORS.

SOUTH BETHLEHEM, PA.,
October 22, 1907.

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INTRODUCTION.

PROCEDURE IN THE LABORATORY AND TREATMENT OF DATA.*

Object of a physical laboratory course. — A physical laboratory course for undergraduate students has a two-fold purpose. On the one hand it serves to illustrate the principles of physics and their application to actual problems, thus acquainting the student at first hand with the phenomena of physics, and giving to him a surer grasp of the principles underlying these phenomena. On the other hand, it is intended to cultivate the power of accurate observation, to familiarize the student with methods of measurement, to give him skill and facility in the use of measuring instruments, and to develop in him the judgment necessary for the making of measurements in a manner adequate to the requirements of science, engineering, and commercial work. A number of somewhat simple and crude experiments has been included in this Laboratory Manual. These experiments do not require any considerable degree of accuracy; they are intended to be purely illustrative. A majority of the experiments, however, are susceptible of a high degree of accuracy, and the student should throughout the course undertake to attain the highest accuracy possible with the material in hand.

The following paragraphs set forth a few simple rules which must be followed in the making of physical measurements and in the obtaining of results from observed data. These rules are entirely essential to the obtaining of trustworthy results and they

*The discussion of treatment of data which is given in this introduction is necessarily very brief. The student will find the following to be the best available discussions of this subject: Kohlrausch, *Physical Measurements*, Leipzig, 1901, pages 1 to 29 (this book is translated into English); Holman's *Precision of Measurements*, John Wiley & Sons, 1897; Merriman's *Method of Least Squares*, John Wiley & Sons, 1884.

should be studied with care before the student begins the experimental work of the laboratory course.

Preparation. — Success in the performance of physical experiments can be expected only when the observer is thoroughly familiar with the nature of the work in hand, the object to be sought, the general theory of the method to be employed, and the apparatus to be used in making the measurements. For this reason, the student must make a careful study of each experiment before entering the laboratory to perform it, and experiments should therefore be assigned in advance.* It is particularly important that the student have in mind just what observations are to be taken, what adjustments made, what sources of error eliminated, and in general what conditions must be satisfied in order that the observations may be trustworthy. This preparation should be such as to make further reference to books unnecessary after work is begun in the laboratory, and to relieve the observer's mind of everything but the work of taking the observations. Blank tabular forms for the record of observations should always be prepared by the student before entering the laboratory. These forms serve as an outline to guide the student in performing his experimental work. Sample tabular forms are given with some of the earlier experiments of this course.

Apparatus. — Accuracy of results depends to a great extent upon the accuracy of construction of apparatus and its suitability to the purpose in hand. In a well-equipped laboratory, proper apparatus will, of course, be provided. The student should, however, acquaint himself with the requirements to be met by

*The authors have found the following system to be quite satisfactory. At the end of a given laboratory period, the experiment which the student is to perform at the next period is assigned; the student is expected to be prepared as stated above, and the thoroughness of his preparation is tested at the beginning of the next period by requiring him to answer in writing a question on some particular detail of the experiment. The tabular form, in which to enter the observations, should be made on a sheet of the paper upon which the final report is to be made so that this data sheet may be bound in with the final report. When the observations are completed this data sheet should be submitted to the instructor in charge for his approval and signature.

the apparatus. This will not only enable him to detect any faults in the apparatus, but also prepare him for its proper use. In all cases care in the handling of apparatus is essential to obtaining the best results.

Reports. — A written report is to be made of each experiment performed.* This report should be written up as soon as possible after the observations have been completed, in order that the details of the work may be fresh in the mind. An adequate report will consist of the following parts, in order.

1. Printed cover-sheet bearing a student's name, the name and number of the experiment, and the date on which the experiment was performed.

2. The original data-sheet; the data always being arranged in tabular form. Averages of data should be struck on this sheet.

3. Full statement of the object of the experiment and of the methods employed.

4.† Description and sketch of apparatus. This description should include the make and number of each piece of apparatus used. The sketch should be diagrammatical rather than pictorial, it should represent the apparatus actually used in the experiment, and it should show the essential features clearly. Non-essential features should be omitted for the sake of clearness.

5. A full statement of the manner in which the observations were taken.

6. Arithmetical work. This is to be indicated by algebraic equations, the manner in which the observed data have been substituted in these equations being clearly shown. These equations, together with other essential parts of the report, should be given on the right-hand pages, and the arithmetical calculations should be placed upon the left-hand pages. It is important that

* The practice of the authors is to require the writing up of this report outside of the laboratory, the report to be handed in at the laboratory period next following that in which the observations were taken.

† This part of the report must be prepared in the Laboratory as a part of the original records of the experiment.

the calculations be preserved in this way for convenience in checking the computations in case of mistake.

7. Tabulation of results, including the averages of observed quantities, probable errors, and other significant quantities.

8. Whenever observations afford data for a plot, a curve should be made.

9. General conclusions.

Observations. — Only original and actual observations are to be recorded on the data sheet. A derived result must not be recorded as an observation, no matter how simple the process of derivation may be. For example, in determining an interval of time the observations consist of a clock reading at the beginning and another at the end of the interval. These clock readings are to be recorded, *not their difference.* There are two reasons for this requirement. Firstly, the giving of the attention to the computation of results would lessen the attention given to *observing.* Results may be computed in the laboratory only when the result is necessary to the completion of the observations. Secondly, a computation made in the laboratory may be incorrect, and if the original data are not preserved, the mistake can never be rectified. For this reason the original data must be preserved, even when a duplicate copy has been made.

Care must be taken to observe exactly what does occur, irrespective of what the observer thinks ought to occur. If the mind is allowed to become prejudiced, large errors of observation may result. Particular care must be taken in the repetition of an observation. The student should try to take the second observation exactly as if the former one had not been taken, or at least as if it had been forgotten. It is only when this has been done that the average of the observed values is the most probable value of the quantity to be measured.

In many cases accuracy of observation requires a certain degree of skill. The student should, therefore, make a preliminary test by performing the adjustments several times and taking the resulting readings. Such readings should, wherever possible, be

made not on the actual thing to be measured in the experiment, but upon something else substituted for it. When such preliminary readings are found to agree fairly well, the actual work of the experiment may be begun.

A single observation is never to be trusted. Repeat the adjustments and observations a sufficient number of times to make the probable error of the result small. Mistakes in readings may frequently be avoided by looking again after the figures have been set down, thus making sure the reading was correctly taken.

Errors of Observation.* — Physical measurements are always subject to error, and means must be found to reduce the effects of these errors as much as possible if accurate results are to be obtained. Errors fall into two general classes, known as *systematic errors* and *erratic errors*. Methods of eliminating these two classes of errors are essentially different.

Systematic Errors. — When a measurement, made with given apparatus under given conditions, is repeated with new apparatus under new conditions, the results differ. This difference is due to constant sources of error inherent in the one set or the other set of apparatus, or in one set or the other set of conditions. Errors of this kind are called *systematic errors*.

A systematic error may sometimes be eliminated by repeating

* Errors of observation may be classified according as they are due to irregularities in the thing which is being measured, or to irregularities in the measuring devices. The former, being inherent in the thing which is measured, may be called *intrinsic errors;* and the latter, being outside of the thing which is measured, may be called *extrinsic errors.* For example, suppose that one makes repeated measurements of the current delivered by an electric-railway power station; the successive measured values of the current will differ greatly from each other and these discrepancies will be due in part to irregularities in the system and in part to irregularities in the ammeter (temperature variations, mechanical shocks, and errors of judgment of the observer in taking the readings). In order to bring out sharply the meaning of intrinsic errors, let us suppose that the extrinsic errors are negligible so that the discrepancies between successive observations may be due wholly to irregularities in the thing which is measured. Thus, the discrepancies between successive readings of current in the above example are due almost wholly to irregularities in the railway system. *Intrinsic errors signify indefiniteness of the thing which is measured, and the probable error of a result, insofar as it depends upon intrinsic errors, is a measure of the degree of indefiniteness of the measured quantity.*

a measurement under conditions which are changed so as to reverse the algebraic sign of the error. A good example of this is the method of double weighing for eliminating the error due to the inequality of the arms of a balance, as described on page 27.

Systematic errors may in some cases be eliminated by calculating their values and applying these calculated values as corrections to the measured value of the quantity. Thus if a steel meter scale is known to be correct at a temperature of 15° C. and if the scale is used at a temperature of 24° C. to measure a given length, then the true length of the scale at 24° C. may be calculated, if the coefficient of expansion of steel is known, and the measured value of the given length may be corrected. The mass of a body as determined by a balance may be corrected for the buoyant effect of the air, if the density of the air, the density of the weighed body, and the density of the weights are known.

The reduction of systematic error to a minimum depends in general upon the use of carefully standardized and calibrated apparatus. Many of the exercises outlined in this laboratory manual are exercises in the standardization and calibration of measuring apparatus. Thus the method of standardizing a set of weights is described in Experiment 17, and the method of standardizing a meter scale is described in Experiment 10.

The standardizing of measuring apparatus is in general a tedious and exacting process, and most physical measurements for technical purposes are carried out with apparatus which has been standardized at the *International Bureau of Weights and Measures* near Paris, at the *National Bureau of Standards* in Washington, or at the *Reichsanstalt* near Berlin.

Personal Errors. — When an observer is free from bias his errors of judgment in the estimation of coincidences and in the estimation of fractions of divisions are to be classed as erratic errors. There are however certain measurements which are affected by systematic personal errors not due to bias or prejudice. Thus the determination by a given observer of the clock reading of a signal by means of the chronograph is subject to a nearly

constant personal error. The clock reading is usually about three quarters of a second too late, the exact amount being different for different observers. This error is due to the time required for the nerve action which is involved in the seeing or hearing of the signal and in the control of the muscles which operate the recording key.

Erratic Errors. — When a measurement is carefully repeated with the same apparatus under constant or nearly constant conditions there are always erratic discrepancies between the successive observations. These discrepancies show the existence of erratic errors. These erratic errors are due to the fact that the circumstances under which a measurement is made are subject to chance variations which are beyond control. These errors are called *erratic errors* inasmuch as they depend upon chance.

Erratic errors are reduced to a minimum by exercising great care in the manipulations involved in a measurement, by taking great care to keep the conditions constant during a measurement, and by taking accurate readings. The effect of erratic errors upon the result of a set of observations is reduced by taking the average of a number of observations.

Example. — A length of several meters is measured by stepping it off by means of two short standards. Such a measurement is affected by erratic errors due to variations of length of the standards caused by irregular and it may be unavoidable changes of temperature, by erratic errors due to unavoidable variations of pressure of the end contacts of the measuring standards, and by erratic errors due to variations of judgment of the observer in estimating coincidences and in estimating fractions of divisions.

Application of the theory of probability to erratic errors. — A measurement having been repeated several times and the resulting observations being found to differ from each other on account of erratic errors, we have the problem of determining (*a*) what is most likely to be the correct value of the measured quantity and (*b*) how nearly correct this value may be supposed to be. Since erratic errors occur by chance, this problem involves the

theory of probabilities. The result of a single observation is as likely to be too large as to be too small, just as a tossed penny is as likely to fall heads up as to fall tails up. Therefore, when an observation has been repeated a number of times, it is most probable that half of the observed values will be too large and half too small; it is also most probable that the sum of the positive errors will be equal to the sum of the negative errors. Consequently, *the average of a number of observations is the most probable value of a measured quantity.*

Probable error. — The average of a number of observations of a measured quantity is, of course, likely to be more or less in error. The probable error, P, of this average is given by the equation

$$P = \pm 0.6745 \sqrt{\frac{S}{n(n-1)}} \qquad (1)a$$

in which n is the number of observations in the set and S is the sum of squares of the differences between single observations and the average. If it is desired to find the probable error, P', of a single observation of a set, we have

$$P' = \pm 0.6745 \sqrt{\frac{S}{n-1}}. \qquad (1)b$$

Example. — A spherometer is repeatedly adjusted into contact with the surface of a true plane, and the following readings are taken (see experiment 8):

Readings.	Residuals (Readings — Average).	Residuals squared.
24.972	— 0.004	0.000016
24.979	0.003	0.000009
24.975	— 0.001	0.000001
24.978	0.002	0.000004
Aver. 24.976		Sum 0.000030

From these data the probable error of the average is found to be

$$P = \pm 0.6745 \sqrt{\frac{0.00003}{4 \times 3}} = \pm 0.001 \text{ approximately,}$$

which means that the actual error of the average, 24.976, is as likely to be greater than 0.001 as it is to be less than 0.001; that there is comparatively little probability that the actual error is as great as 0.002, and still less probability that it is as great as 0.003. Thus the probable error of a result is an estimate of the limits between which the actual error of a measured quantity may be safely supposed to lie, provided the measurement has been affected by erratic errors only.

Combination of erratic errors.— When several observed quantities enter into a result it is very improbable that the errors in the respective observed quantities should all conspire to produce errors of the same sign in the result. The probable error of the result is determined as follows: Let x, y, z, etc., be the observed quantities each determined by a set of observations, and let P_x be the probable error of x [see equation (1)], P_y the probable error of y, P_z the probable error of z, etc. The result r to be calculated is always a known algebraic function of x, y, z, etc., so that we may write

$$r = F(x, y, z \cdots) \qquad (2)$$

where F represents a known function. The desired probable error P_r of the result is then found from the equation

$$P_r = \sqrt{\left(P_x \cdot \frac{dr}{dx}\right)^2 + \left(P_y \cdot \frac{dr}{dy}\right)^2 + \left(P_z \cdot \frac{dr}{dz}\right)^2 + \cdots} \qquad (3)$$

Examples.—(1) The simplest example is that in which the desired result r is the sum or difference of two readings x and y, that is, when $r = x \pm y$. In this case dr/dx and dr/dy are both equal to unity, algebraic signs being ignored, so that equation (3) becomes $P_r = \sqrt{P_x^2 + P_y^2}$. That is, *the probable error of the result is equal to the square root of the sum of the squares of the probable errors of the respective sets of readings.*

(2) As a second example suppose that the length x and the breadth y of a rectangle have each been measured a number of times, suppose that P_x is the probable error of x and P_y the

probable error of y, and suppose that the desired result is the area of the rectangle so that $r = xy$. In this case equation (3) becomes

$$P_r = \sqrt{P_x^2 y^2 + P_y^2 x^2}.$$

Rule for finding the probable error of a result without the use of calculus. Let x, y, and z be the values of the measured quantities, each the average of several observations, and let P_x, P_y, and P_z be the probable errors of x, y, and z respectively. All of these quantities are known. (*a*) Calculate the value of the result from x, y, and z; (*b*) Calculate the result from $(x \pm P_x)$, y, and z; (*c*) Calculate the result from x, $(y \pm P_y)$ and z; (*d*) Calculate the result from x, y, and $(z \pm P_z)$; (*e*) Subtract the result (*a*) from each of the results (*b*), (*c*), and (*d*) and take the square root of the sum of the squares of these remainders [= the probable error of the result (*a*)].

Significant figures. — In the above example of spherometer readings, the average reading is 24.976 and the probable error of this result is ±0.001, so that the last figure in 24.976 is untrustworthy. The probable error of a result affords a basis for deciding how many significant figures should be carried out in the calculations which are based upon a set of observed data. By significant figures is meant all figures except cyphers used to show the position of the decimal point. Thus in each of the numbers 20500 and 0.00205 there are three significant figures, the 205, only, being counted. The average value 24.976 of the above set of spherometer readings contains five significant figures; only four of these, however, are trustworthy, and no result computed from this value can be trustworthy beyond the fourth significant figure, no matter how accurate the other factors in the computation may be. It is customary to carry throughout a calculation one more figure than is trustworthy.

In the taking of observations and in the computation of results this matter of the number of significant figures must be kept in mind. Suppose, for example, that in performing a certain ex-

periment, two different quantities are to be measured, and suppose that these two quantities enter equally into the computation of the result. If one of them can be measured only to three significant figures, it is useless to carry the measurement of the other beyond four figures. If, however, the first power of the one quantity and the second power of the other quantity are involved in the computation of the result, the second quantity should be measured with the greater accuracy, inasmuch as its error has a greater effect upon the result. Furthermore, if the observed quantities are correct to five figures, it would be folly to use in the computations a constant which is correct to only three or four figures. For example, the value $\frac{22}{7}$, which is often used for π, is correct to three figures only, and this value is therefore not sufficiently accurate where data have been determined to four or more figures.

Treatment of data. — The conditions under which measurements are made, especially in engineering tests, are extremely varied, and no general directions can be given for working up a final result from repeated sets of observations. The following examples must therefore serve as a general guide.

(*a*) *Case in which the conditions remain sensibly constant during the time that measurements are being made.* For example, the length and diameter of a cylinder are repeatedly measured and it is desired to determine the volume of the cylinder. In this case, the average of each measured quantity is found and the volume is calculated from the average length and the average diameter. The probable error of the average length and the probable error of the average diameter are found and the probable error of the result is calculated as explained on page 10. In this case, the discrepancies between repeated measurements are due almost wholly to variations in the measuring devices and the probable error of the result is a measure of its accuracy.

(*b*) *Case in which the conditions vary perceptibly during the time that measurements are being made.*

Example 1. — In determining the efficiency of a water motor,

the amount of energy delivered to the motor by the water during a given run is to be determined as accurately as possible in spite of irregular variations of the water pressure which are beyond the control of the observer. During a five-minute run, 25.42 gallons of water pass through the motor and the pressure of the water supply is observed at intervals of one-half minute as follows: 64.5, 66.2, 65.1, 65.8, 65.0, 65.4, 65.7, 65.3, 65.9, 65.1, 65.3 pounds per square inch. In this case, the best procedure is to take the average of the observed pressures, namely, 65.44 pounds per square inch, which, multiplied by the volume of water in cubic inches, gives the work that is delivered to the motor, namely, 384,300 inch-pounds.

In general, when the conditions which prevail during a test are erratically variable, as in this example, the average of each set of observed quantities should be taken and the results should be calculated from these averages.

When the conditions which prevail during a test are sensibly constant, the probable error of the result is a measure of the accuracy of the observations. When the conditions which prevail during a test are erratically variable, the probable error of the result may be considered to be a measure of the accuracy of the result; but in strictness, it is a measure of the definiteness of the thing which is being measured, inasmuch as the errors are due to variations in the thing which is being measured and not to variations in the measuring devices.

Example 2.—A vessel of hot water is allowed to cool and simultaneous readings of clock and thermometer are taken as indicated in the accompanying table.

Temperature of vessel.	Elapsed Time.
87° C.	0 min.
70	5
62	10
55	15
47	20
44	25
39	30
35	35
31	40

Room temperature 18.0° C.

The individual observations of this set are shown by the points in Fig. 1 and the smooth curve is the most accurate approximation to the actual relation between the temperature of the cooling vessel and the elapsed time. It is evident that the actual temperature of the vessel at a given instant may be read off the curve in Fig. 1 with much greater accuracy than it can be determined by taking the average of any set of thermometer readings.

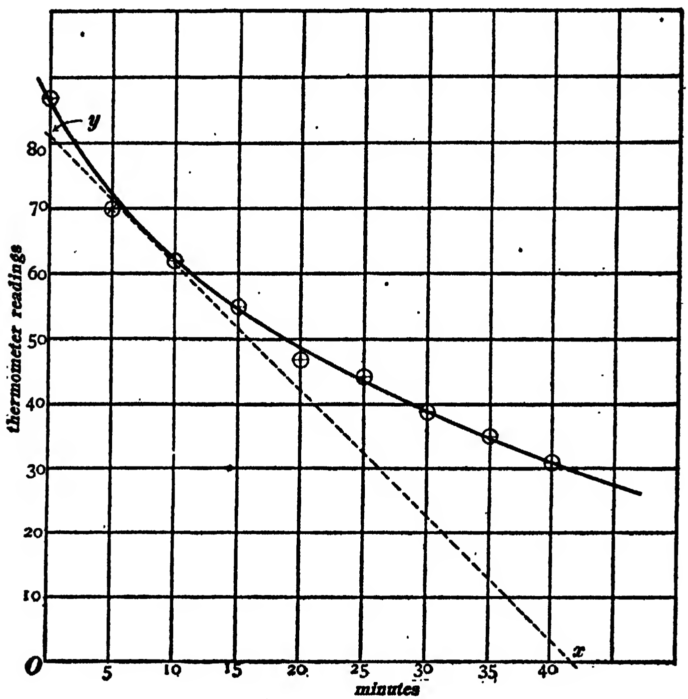


Fig. 1.

When a curve has been plotted to represent the relation between two variable quantities, as in Fig. 1, the slope of the curve at a given point represents the rate of change of the one variable with respect to the other at that point. The value of this rate of change is determined by drawing a tangent to the curve at the given point and dividing the intercept Oy by the intercept Ox (see Fig. 1). Thus the rate of change of the temperature of

the vessel of hot water in the above example at the instant $7\frac{1}{2}$ minutes after the first observation was taken, was 81° (= Oy) divided by 42 minutes (= Ox), or 1.93 degrees per minute.

In general, when the conditions which prevail during a test vary according to some definite law, the individual observations should be plotted on cross-section paper, a smooth curve should be drawn as in the above example, and the desired result should be calculated from the ordinates of any chosen point on the smooth curve.*

The errors (residuals) of the individual observations in Fig. 1, for example, are the vertical distances of the plotted points from the smooth curve and the probable error of a single observation may be determined from these residuals by using equation (1b).

Further use of plotted curves. — The use of plotted curves for determining (a) the value of a varying quantity at a given instant or (b) the rate of change of a varying quantity at a given instant, is explained above. Another use of the plotted curve, which indeed is essentially the same as (a), is as follows: Suppose that two related quantities x and y are observed. For example one end of a bar of metal may be in a flame and y may be the temperature of the bar at a point distant x from the end. By plotting each pair of observed values of x and y as a point of which x is the abscissa and y is the ordinate, a *smooth curve* may be drawn among the plotted points and this smooth curve will usually represent the best approximation to the true relation between x and y. In Experiment 5 is a good example of this use of a plotted curve.

The plotting of curves. In the plotting of a curve from experimental data it is quite necessary to use accurately ruled cross-section paper. Errors of ruling produce sharp bends in a curve plotted from accurate data, and they produce irregularities in the values of ordinates and abscissas which are read off from a

* The application of the method of least squares to this particular problem is discussed in Kohlrausch's *Physical Measurements*, English edition page 21; ninth German edition, page 19.

smooth curve. The plotted points should always be clearly marked, as in Fig. 1 for example, and the scales of abscissas and of ordinates should be chosen so as to show the irregularities (the erratic errors) of the observed quantities distinctly. This is exemplified by the directions given for plotting the barometric curve in Experiment 3.

PROBLEMS.

1. A spherometer was adjusted 10 times in succession so as to bring the screw-point into contact with a true plane, and the readings A in the following table were obtained. The spherometer was then adjusted 10 times so as to bring the screw-point into contact with a small glass block resting on the true plane and the readings B in the following table were taken. Find:

A	B
2.01	23.47
2.26	23.38
2.23	23.83
2.06	23.32
2.13	23.54
2.08	23.57
2.34	23.52
2.00	23.09
2.64	23.38
2.43	23.47

(*a*) The mean of each set of readings and the probable error of each mean; (*b*) take the difference between the two means and find the probable error of this difference as explained in example 1 on page 9; (*c*) take the difference between the various pairs of readings B-A, find the mean of these differences, and find the probable error of this mean. Ans. (*a*) Mean of A readings, 2.22, probable error, 0.0432, mean of B readings, 23.46, probable error, 0.0408; (*b*) difference between two means is 21.23, probable error of this difference 0.0594; (*c*) mean value of differences 21.23, probable error of this mean, 0.0549.

Note.—The results (*b*) and (*c*) in this problem would approximate to exact equality if the number of observations were very large.

2. The length and breadth of a rectangle are measured repeatedly giving the following values:

Length.	Breadth.
68.45 cm.	32.53 cm
68.48	32.51
68.47	32.52
68.44	32.50
68.46	32.54

Find the probable error of the computed area of the rectangle. Ans. ± 0.35 square centimeter.

3. The length and diameter of a small cylinder were measured repeatedly giving the following values:

Length.	Diameter.
1.987 in.	0.529 in.
1.986	0.532
1.989	0.530
1.985	0.531
1.983	0.531

Find the probable error of the computed volume of the cylinder. Ans. 0.265 millionth of a cubic inch.

4. The force required to drag a wooden block slowly across a smooth table was observed repeatedly, giving the following values in ounces: 56, 52, 59, 55, 58, 53. Find the probable error of a single observed value of this set. Ans. ± 1.686 ounces.

Note. — The thing which is measured in this case is inherently erratic and the discrepancies between the observed values are due not so much to inaccurate measurement as to inherent variability of the thing measured. Therefore the probable error in this case is an indication of the indefiniteness of the measured quantity. See footnote on page 5. When the errors due to erratic variations in the measuring apparatus (extrinsic errors) are negligibly small in comparison with the errors due to erratic variations in the measured quantity (intrinsic errors), the probable error should perhaps be called the *probable departure* of the measured quantity from the mean of the observed values.

5. A window in a room was thrown open and a thermometer in the room gave the following readings at intervals of 30 seconds: 16.22, 15.45, 14.95, 14.53, 14.30, 13.90, 13.60, 13.42, 13.20, 13.04, 12.91, and 12.90. Plot a curve of which the abscissas represent elapsed time and ordinates the changing temperature of the room. Draw a smooth curve through the plotted points

and determine: (*a*) the rate of change of the temperature at the instant of the fifth reading; (*b*) the approximate errors of the various readings. Ans. (*a*) 0.633° per minute; (*b*) the approximate values of the errors are as follows in the order of the readings: + 0.02, − 0.04, 0, − 0.01, + 0.10, − 0.01, − 0.04, + 0.02, − 0.005, 0, − 0.01, and + 0.04.

Note. — The approximate errors of the various readings are the vertical distances between the plotted points and the smooth curve. These errors are almost wholly due to erratic variations of the thing which is being measured and not to inaccuracies in the measuring apparatus.

This problem exemplifies a case which occurs very frequently, in which a quantity which is being measured is subject to two distinct kinds of variations, a systematic variation and an erratic variation. The systematic variation is shown by the general trend of the curve and the erratic variations are shown by the departures of the plotted points from a smooth curve. It will be noted that the sum of the errors as given in the above answer is not equal to zero. This is on account of the difficulty of drawing a smooth curve through the plotted points so as to represent the plotted points in the best possible way; and of course the student cannot expect to get exactly the same set of values for the errors as given in the above answer.

PART I.

MEASUREMENT OF LENGTH, ANGLE, MASS AND TIME.

LIST OF EXPERIMENTS.

1. Practice with the simple vernier. Estimation of fractions of divisions by the eye.
2. The vernier caliper.
3. The barometer.
4. The cathetometer.
5. Study of a divided circle.
6. The reflecting goniometer.
7. The micrometer caliper.
8. The spherometer.
9. The micrometer microscope.
10. The comparator.
11. The dividing engine.
12. Base-line measurement.
13. Measurement of length by means of the reading telescope.
14. The optical lever.
15. Weighing by swings. Elimination of errors.
16. The sensibility of the balance.
17. Errors of a set of weights.
18. Determination of the volume of a flask by weighing.
19. Determination of the density of a liquid.
20. Determination of the density of a solid.
21. Study of the rate of a watch.
22. The eye and ear method of observing time intervals.
23. The chronograph.
24. The determination of gravity. Method of coincidences.

INSTRUMENTS FOR THE MEASUREMENT OF LENGTH.

The scale and vernier. — A length is usually measured by applying to it a divided scale, and noting the readings on the scale which correspond with the ends of the length. For example, Fig. 2 shows a scale applied to a rod, the length of which is to be measured. The reading of the scale at one end of the rod is 5.2 centimeters and the reading of the scale at the other end of the rod is 12.6 centimeters, the fractions of a division being estimated by the eye as 0.2 and 0.6 centimeters respectively. The length of the rod is then found by taking the difference between these two readings, namely $12.6 - 5.2 = 7.4$ centimeters.

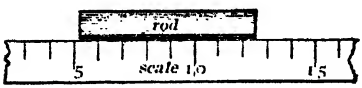


Fig. 2.

In applying the scale for the measurement of a length, it is very important to bring the edge of the scale into actual contact with the points which mark the ends of the length to be measured. This is often impossible, and therefore the accurate measurement of a length usually requires some sort of a sighting device like the microscope or telescope. For example, the comparator (see Experiment 10), makes use of the microscope for sighting at the ends of the length to be measured, and the cathetometer (see Experiment 4) makes use of the telescope for sighting at the ends of a length to be measured.

The very considerable errors involved in the estimation of fractions of divisions in the use of divided scales are to a great extent obviated by the use of the vernier. The vernier consists of a short auxiliary scale V, Fig. 3, divided into n equal parts, each division on the vernier being $1/n$th shorter than the divisions on the main scale, which we will call millimeters for brevity. Figure 3 is constructed for $n = 10$. Let the space f

be the fraction of a division to be determined and suppose it is equal to $\frac{7}{10}$ of a millimeter; the space g is $\frac{1}{10}$ of a millimeter shorter than f, the space h is $\frac{2}{10}$ of a millimeter shorter than f, and so on, so that the 7th mark on the vernier must be coincident with a mark on the scale. *The number of the mark on the vernier which is coincident with a mark on the scale is the* **numerator** *and the number of divisions on the vernier is the* **denominator** *of the fraction which expresses the space f in terms of a scale division.*

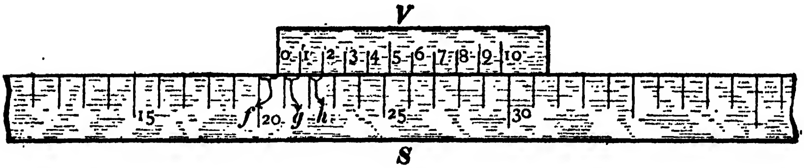


Fig. 3.

The position on the scale of the zero mark of the vernier is called the *reading of the vernier.* Thus, the reading of the vernier in Fig. 3 is 20.7.

The vernier is always provided with a reference line which is made to coincide first with one end and then with the other end of the object to be measured; and the difference between the two vernier readings is equal to the length of the object. For example, the face of the movable jaw of the vernier caliper (see Experiment 2) is the reference line with respect to which measurements are made. Thus a vernier caliper reads $0\frac{2}{20}$ of a millimeter when the jaws are together, and it reads $74 + \frac{13}{20}$ millimeters when the caliper jaws are in contact with a cylinder, the diameter of which is to be measured, so that the diameter of the cylinder is $(74 + \frac{13}{20}) - \frac{2}{20} = 74.55$ millimeters. In the cathetometer, the axis of the sighting telescope is the reference line (see Experiment 4).

The vernier is also used with the divided circle for the measurement of angle. For example, a surveyor's transit has a circle divided to thirds of a degree, and a vernier of twenty parts. This

vernier reads to $\frac{1}{20}$ of $\frac{1}{3}$ of a degree which is equal to one minute. In this case, the axis of the telescope which is mounted on the alidade constitutes the reference line which moves with the vernier.

The micrometer screw. — The micrometer screw consists of an accurately-cut screw to which is attached a circular head, the circumference of which is divided into equal parts. The micrometer screw is used for the measurement of a length in terms of the distance between the threads of the screw taken as unity, fractional parts of this distance being indicated by the divisions on the circular head. A straight scale is usually provided, from which whole turns of the screw may be read off, this scale being divided into parts equal to the pitch of the screw.

Example. — The micrometer caliper is the most familiar example of the micrometer screw. The micrometer caliper is arranged so that the reading of the screw gives the distance between the caliper tips directly, whole turns being indicated by the scale T, and fractions of a turn being indicated by the scale F, see Fig. 4. In Fig. 4 the pitch of the screw is $\frac{1}{20}$ of a

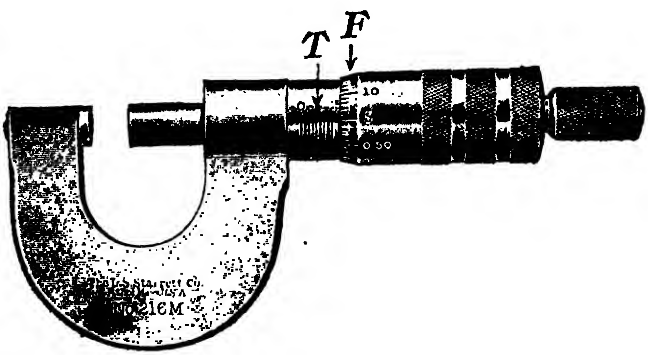


Fig. 4.

centimeter, the scale of turns is divided into parts each $\frac{1}{20}$ of a centimeter long, and these divisions are numbered for convenience as centimeters, millimeters, and half-millimeters. The circular head of the screw is divided into fifty parts, each of which represents $\frac{1}{50}$ of $\frac{1}{20} = 0.001$ centimeter. Sometimes a vernier is

attached to the circular head of the micrometer screw to indicate fractions of divisions.

The micrometer screw is an essential feature of a variety of measuring instruments. Thus, a micrometer screw is used to move the graving tool of a dividing engine in the manufacture of scales (see Experiment 11), the micrometer screw is used to move the cross-hairs of the micrometer microscope (see Experiment 9).

In using a measuring instrument which has a micrometer screw, care must be taken to avoid errors due to lost motion of the screw. *The setting of the instrument must always be made by turning the screw in a particular direction so that the movable part is always pushed into position from the same side.*

THE MEASUREMENT OF MASS.

The balance.* The result of the weighing of a body by means of a balance is called the *mass* of the body. The balance consists of a delicately mounted equal-arm lever with pans suspended from its ends. *The balance is used simply for indicating the equality of the masses of two bodies, and the determination of the mass of a body by means of the balance depends upon the use of a set of weights which may be combined in such a way as to match the mass of the body.*

A slender *pointer* which is attached to the balance beam plays over a scale at the bottom of the supporting column and serves to indicate the displacement of the beam from its normal position. A device called the *arrestment* is provided by means of which the beam and pans may be lifted in order to relieve the knife edges, and a device called the *pan stop* is provided by means of which the oscillations of the balance beam may be controlled. The arrestment and the pan stop are operated by a milled head outside and in front of the balance case.

The following rules must be faithfully followed in the use of

* The student is referred to the article *Balance* in the Encyclopædia Britannica for a full discussion of the construction and theory of the balance.

the balance in order to protect the balance and weights from injury and to secure accurate results.

1. Always raise and lower the arrestment and operate the pan stop slowly and gently.

2. Always relieve the knife edges by raising the arrestment whenever anything is to be placed upon or taken off the pans. In general, keep the balance lifted off the knife edges except when observing the swings of the pointer.

3. Never allow anything damp to touch the pans, and do not touch them unnecessarily with the fingers. Avoid pendulous motion of the pans while reading the swings of the pointer. Pendulous motion may be stopped and the range of the swings of the pointer may be controlled by cautiously raising and lowering the arrestment or by manipulating the pan stop.

4. The balance case must be kept closed while readings of the pointer are being taken in order to avoid air currents. Never weigh anything hot on the balance. The presence of a hot body in the balance case produces air currents and causes unequal expansion of the various parts of the balance beam, both of which vitiate the observations.

5. Weights must be handled with tweezers, never with the fingers. The small weights always have a corner turned up to form an ear by which they may be lifted. Be careful to keep these small weights right side up in order that this ear may be readily caught by the tweezers, otherwise the small weights are likely to be broken in handling.

6. In matching the mass of a body, try the weights in the order of their magnitude, beginning with the large weights. If weight A is too heavy, try B; if B is too light, leave it on the pan and try C; and so on.

7. Always read off and record the values of the weights before removing them from the pan. The value of each weight is to be recorded. When there are two or more weights of one denomination, note which one is used in order that corrections may be made for the errors of the weights. Different weights

of the same denomination are distinguished by prime marks. Thus, in a set which contains three one-gram weights, the marks 1, 1′, and 1″ are stamped upon them. Each weight must be in its proper place in the box when not actually in use.

Weighing by swings.—A body of unknown mass B is placed upon one pan of the balance and counterpoised as nearly as possible by weights of known mass W placed on the other pan. If it were possible to exactly counterpoise the body B, the only errors in the use of the balance would be the systematic errors due to inequality of balance arms and to the buoyant effect of the air; but it is not possible to exactly counterpoise a body which is being weighed; that is to say, there is always a slight difference between the values of B and W* and we may write

$$B = W \pm a$$

The problem of weighing is to determine the value of this difference a. (1) The position of rest N of the pointer with empty pans is observed, (2) the position of rest P_1 of the pointer when

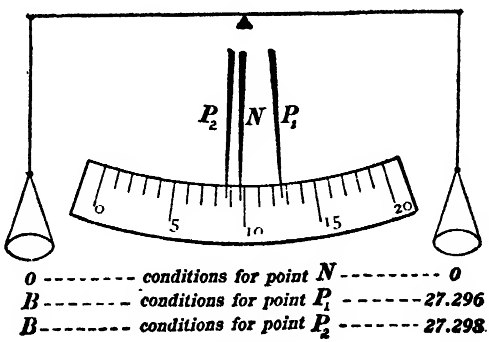


Fig. 5.

the loads B and W are placed upon the pans is observed, and (3) the position of rest P_2 of the pointer is observed when an additional known weight w is placed upon one of the pans, as shown in Fig. 5. The movement of the position of rest from

* Errors due to inequality of arms and to buoyant affect of the air are not considered for the present.

P_1 to P_2 is due to the known weight w, and the amount of weight a' which should be added to the one or the other of the pans to bring the position of rest of the pointer to the normal position N (that is, to exactly counterpoise the body), is equal to the fractional part $(P_1 - N)/(P_1 - P_2)$ of w.

The scale upon which the position of the pointer is read has usually 20 divisions and these divisions should be numbered from the left end of the scale. The normal position N of the pointer will be near the 10th division if the balance is in proper adjustment. The positions of each of the three points N, P_1, and P_2 is determined as follows: The balance is set swinging, and an odd number of successive extreme positions of the moving pointer is observed. The average of these readings, taken as shown below, gives the required position of rest, N, P_1, or P_2 as the case may be.

Example.—(1) The balance case is closed, the arrestment is lowered, the pan stop is manipulated so as to set the balance beam swinging through a small amplitude, and the following readings are taken in order.

Left.	Right.	
1- 3.9	2-15.6	
3- 4.0	4-15.5	4.10
5- 4.2	6-15.3	15.47
7- 4.3	3/46.4	2/19.57
4/16.4	15.47	9.78 = N
4.10		

(2) The body is then placed upon the left pan and weights equal to 27.296 grams are placed upon the right pan, the balance case is closed, the beam is set swinging as before, seven successive elongations of the pointer are observed, and the position of rest is found to be $13.6 = P_1$.

(3) A weight of two milligrams (0.002 gram) is then added to the right pan, making the total weight equal to $(27.296 + 0.002)$ grams, the balance case is closed, the balance is set swinging, and seven successive elongations of the pointer are observed as before, from which we find the position of rest to be $8.7 = P_2$.

From these observations we find by the method above explained:

$$B = 27.296 + \frac{13.6 - 9.78}{13.6 - 8.7} \times 0.002 = 27.29703 \text{ grams.}$$

Correction for inequality of arms.*—The error due to inequality of balance arms may be eliminated by weighing the body first on one pan and then on the other pan, using the above method of weighing by swings. Let W_r be the weights required on the

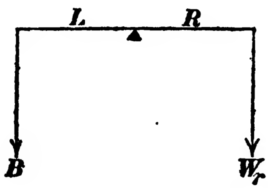


Fig. 6.

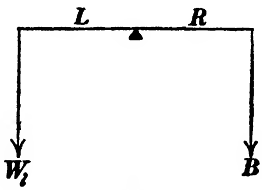


Fig. 7.

right pan to balance the body B on the left, and let W_l be the weights required on the left pan to balance the body B on the right. Then

$$B = \sqrt{W_r W_l} \tag{4}$$

in which B is the mass of the body unaffected by errors due to inequality of arms. In practice W_r and W_l are always very nearly equal to each other, and the square root of their product is almost exactly equal to one half their sum, so that instead of equation (4) we may use the following equation without appreciable error:

$$B = \frac{W_r + W_l}{2} \tag{5}$$

* The data here specified for the elimination of the error due to inequality of arms may be used to calculate the ratio of the lengths of the arms as follows: According to the principle of the lever, we have the following equations from Figs. 6 and 7:

$$BL = W_r R$$

and

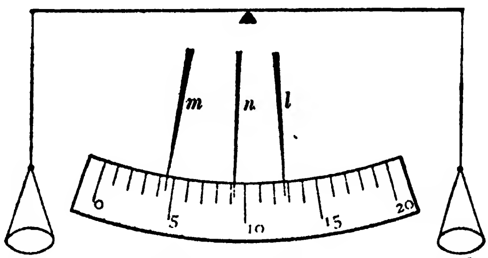
$$BR = W_l L$$

from which we find

$$\frac{R}{L} = \sqrt{\frac{W_l}{W_r}}$$

The above method of eliminating the errors due to inequality of the balance arms requires the taking of six sets of swings, three sets for determining W_r and three sets for determining W_l. The following method, known as the *method of double weighing*, accomplishds the same result without appreciable error, and requires only three sets of swings to be taken.

1. Place the body B to be weighed on the left pan, balance it by means of very nearly equal weights W on the right pan, and take a set of swings for determining the position of rest of the pointer, as explained above. Let l be the position of rest of the pointer so determined. See Fig. 8.



B--------conditions for point l--------W
W--------conditions for point m--------B
W+w--------conditions for point n--------B

Fig. 8.

2. Interchange B and W, take a second set of swings, and let m be the position of rest of the pointer so determined.

3. Place a small additional known weight w on the left pan, making the total weight on the left pan equal to $W+w$, and take a third set of swings. Let n be the position of rest of the pointer so found

If B and W were exactly equal to each other they would of course not balance exactly if the arms were unequal, ***but the position of the rest of the pointer would be unchanged when B and B are interchanged***, that is, l would be the same as m in Fig. 8. Therefore the distance from l to m is the movement of the position of rest of the pointer due to $2a$, where a is the difference between B and W. Since $l-m$ is the displacement

of the pointer produced by $2a$, and $n-m$ is the displacement of the pointer due to the known small weight w, we have

$$\frac{2a}{w}=\frac{l-m}{n-m} \tag{iv}$$

from which a becomes known so that

$$B=W\pm a \tag{v}$$

The sign in this equation is to be determined as follows: *The movement of the position of rest when B and W are interchanged is in the direction away from the pan on which the heavier of the two (B or W) is placed after the interchange.**

Correction for buoyancy of air. — When the weighed body is of the same material as the weights, the buoyancy of the air introduces no error. When the weighed body is more bulky than the weights, that is, when the weighed body is of smaller density than the weights, the effect of the buoyancy of the air is to cause an underestimate of the mass of the body; that is, the value of B as above determined is too small. When the weighed body has a greater density than the weights, the value of B as above determined is too large. The true mass M of the body is given by the equation

$$M=B-\frac{B\lambda}{\delta}+\frac{B\lambda}{\Delta} \tag{vi}$$

in which λ is the density of the air, δ is the density of the weights, and Δ is the density of the body. The density of brass weights is 8.4 grams per cubic centimeter, the density of air is approximately 0.0012 gram per cubic centimeter, and the density of the body must be found from tables.

Corrections for errors of weights. — In making accurate weighings it is always necessary to apply corrections for errors of the weights used. The errors of the weights are determined by the

* The method of double weighing is really a method for actually weighing the difference a between B and W, and, inasmuch as this difference is always very small, the inequality in the arms need not be considered. The error due to the inequality of arms would be but a very small fraction of the very small weight a.

method described in Experiment 17, and a table of the corrected values of the individual weights should be placed in the cover of the box in which the weights are kept.

THE MEASUREMENT OF TIME.

An interval of time is measured by observing the readings of a clock at the beginning and at the end of the interval, the difference between the two readings giving the value of the interval. Measurements of time intervals are ordinarily made to determine the period of vibration of an oscillating body, or the speed of rotation of a machine, or any rate of change, such as the rate of deposition of a metal by the electric current. These determinations may be made in two ways (*a*) the interval of time may be arbitrarily chosen and the number of oscillations, or revolutions, or the amount of change during the interval, observed; or (*b*) an arbitrary number of oscillations, or revolutions, or an arbitrary amount of change may be chosen, and the corresponding interval of time observed. In some cases the first method only is feasible, in some cases the second method only is feasible, and in some cases either may be used. For example, the rate of deposition of a metal by an electric current is best determined by allowing the electric current to flow during a chosen interval of time and weighing the deposit. The speed of a runner is best determined by observing the time required to run a chosen distance. The periodic time of a vibrating body may be determined either by counting the number of vibrations in a chosen interval of time, or by observing the time interval required for a chosen number of vibrations.

It frequently happens that a single observer cannot note the signal which marks the beginning or end of a time interval and take the clock reading at the same time. Therefore in many cases, two observers must coöperate in making time observations, as in the following examples:

(*a*) It is desired to determine the number of revolutions of a

machine during a chosen interval of time. One observer, looking at his watch, gives a sharp signal at the beginning and again at the end of the chosen interval. At the first signal the other observer promptly applies the revolution counter to the shaft of the machine, and at the second signal he promptly detaches the counter from the shaft. The signals should be given by saying "Ready — Now," the "ready" being spoken about one second before the desired instant, and the "now" being spoken as sharply as possible at the desired instant.

(*b*) It is desired to determine the period of a slowly vibrating pendulum. In this case, the above procedure should be reversed; that is, one observer watching the pendulum, gives a signal at the beginning and another at the end of a count of a chosen number of vibrations, and the other observer takes the clock reading corresponding to the two signals.

(*c*) It is desired to determine the error of a watch. If the beats of the standard clock can be heard distinctly, a single practiced observer can determine the error of the watch with greater accuracy than two observers; but when the beats of the clock cannot be heard, one observer should look at the clock and give a signal (with warning) at a chosen clock reading, and the other observer should note the corresponding reading of the watch.

The method for taking clock readings when an observer can hear the beats of the clock distinctly and see the signal is called the eye and ear method. The observer glances at the clock, noting the hour, minute and second, and as he looks for the signal which marks the beginning or end of the interval to be measured, he continues to count seconds by listening to the beats of the clock and he estimates the exact clock reading at the instant of the signal. See Experiment 22.

The chronograph is an instrument for enabling clock readings to be taken with greater ease and accuracy than is possible by eye and ear. A pen traces a line on a uniformly moving strip of paper. This pen is fixed to the armature of an electromagnet

which is excited at each beat of the clock by an electric current controlled by a contact device on the clock pendulum. A kink is thus made in the traced line at each beat of the pendulum. At the instant for which the clock reading is desired the electromagnet is momentarily excited by pressing a key which closes an auxiliary electric circuit, thus making an extra kink in the line; and the clock reading is determined by measuring off the extra kink among the kinks produced by the beats of the pendulum. This description applies to the essential features of the chronograph. In the form of instrument as ordinarily used for accurate time observations, a sheet of paper is wrapped around a cylinder which is rotated at uniform speed by clockwork. The tracing pen and electromagnet are mounted on a sliding carriage which is moved slowly parallel to the axis of the rotating cylinder by a screw which is driven by the same clockwork which drives the cylinder. The pen thus traces a helical line on the *paper-covered cylinder, so that the large sheet of paper is equivalent to a very long strip.*

When it is desired to determine the periodic time of vibration of a pendulum which oscillates very nearly in the same rhythm as the clock pendulum, the *method of coincidences* so-called may be used. This method is fully described in Experiment 24.

EXPERIMENT I.

Practice with the Simple Vernier. Estimation of Fractions by the Eye.

The object of this experiment is to familiarize the student with the use of the vernier, and to afford practice in the estimation of fractions of divisions.

The apparatus consists of a metal scale and vernier. The vernier has an auxiliary mark *b*, Fig. 9, etched upon it a short distance from its zero mark. The divisions of the scale should be about one half inch, and the vernier should read to 25ths or 32ds of a division.

Work to be done. — Set the vernier so that the *b* mark is near to one end of the main scale. Read the vernier and read the position of the *b* mark, estimating the fractions in tenths. The vernier reading should be recorded exactly as read. Thus the estimated reading of the *b* mark in Fig. 9 is 5.3 and the reading of the vernier is 7 and 7/25. When one pair of readings has

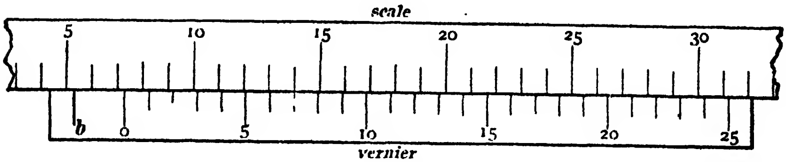


Fig. 9.

been taken, move the vernier a short distance at random, closing the eyes or fixing them on some distant object so as to avoid deliberate setting of the *b* mark, and again read the vernier and estimate the reading of the *b* mark. Proceed in this manner until twenty or more readings have been taken. Tabulate the readings as indicated in the accompanying tabular form.

	Vernier Readings.	Reduced Vernier Readings.	Estimated Readings of *b* Mark.	Differences.	Errors.
1	....	....	....	....	....
2	....	....	....	....	....
3	....	....	....	....	....
Etc.	....	....	....	....	....

Computations and results. — Reduce the vernier readings to decimals, and subtract from each reduced vernier reading the corresponding reading of the *b* mark. The differences thus found are observed values of the distance *d* between the *b* mark and the zero on the vernier, the errors of these observed values being due almost wholly to the errors in the estimated readings of the *b* mark. It is desired to find the approximate values of these errors. Find the average of the observed values of *d* and sub-

tract this average value from each observed value. This gives the approximate values of the errors. Determine the probable error of the average of the observed values of *d*, and find the probable error of a single observed value of *d*.

EXPERIMENT 2.

The Vernier Caliper.

The object of this experiment is to afford practice in the use of the vernier caliper.

Apparatus. — The vernier caliper is shown in Fig. 10. If the

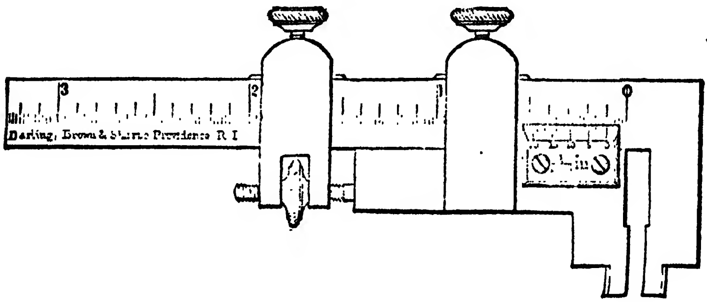


Fig. 10.

zero of the vernier coincides with the zero of the scale when the caliper jaws are closed, then, when the jaws are opened, the reading of the vernier gives the correct distance between them. It frequently occurs, however, that the vernier reading is not zero when the caliper jaws are closed, so that a correction must be applied to any given reading of the vernier to give the true distance between the caliper jaws. This correction is equal to the reading of the vernier when the jaws are tightly closed. This is called the *zero error* of the caliper, and it is easily determined by inspection whether it is to be added to or subtracted from the vernier reading to give the correct size of an object placed between the jaws.

Work to be done. — It is desired to determine the mean length, mean diameter, and volume of each of several short metal rods.

Determine the zero error of the caliper, and decide whether it is to be added to or subtracted from subsequent readings of the vernier. Indicate this decision by marking the zero error with the positive or negative sign.

To determine the length of the rod, set the rod lengthwise between the jaws of the vernier as shown in Fig. 11, read the vernier, then turn the rod through 90° about its axis, set the jaws and read the vernier again. Proceed in this way until readings have been taken for four successive positions of the rod.

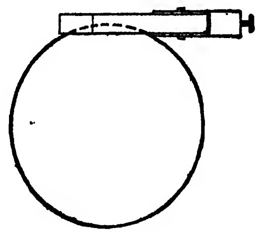


Fig. 11.

To determine the mean diameter of the rod, take readings of the diameter at five or six equidistant points along the rod, and at each point measure two mutually perpendicular diameters.

Computations and results. — Average the readings of length, and the readings of diameter, and determine the probable errors of these averages. Correct the averages for zero error and compute the volume of each rod from the average length and average diameter. Find the probable error of the calculated volume of each rod.

EXPERIMENT 3.

The Barometer.

The object of this experiment is to determine the average barometer reading during a given period of time, and to apply the various corrections for systematic errors. The barometer curve for the given period is also to be plotted.

Apparatus. — Figure 12 shows the very convenient form of barometer cistern which is due to Fortin. The barometer scale is etched upon the brass containing tube, and the scale is adjusted so that the scale indicates centimeters or inches from the tip of the ivory point *r*. Therefore, when the surface of the mercury in the cistern is brought into contact with the ivory point *r*, the

scale reading indicates the height of the mercury column which is balancing the atmospheric pressure.

Work to be done. — A few preliminary observations should be taken in order that the student may become familiar with the adjustments, and with the method of reading the barometer.

Bring the surface of the mercury in the cistern barely into contact with the ivory point *r*, Fig. 12, by turning the screw *S* at the bottom of the cistern. Then tap the barometer gently with the finger in order to bring the meniscus to its normal shape. *This adjustment must be repeated before every reading and the ivory point r should be viewed through a magnifying glass.*

Then raise or lower the slider SS, Fig. 13, by means of the milled head at the side of the barometer tube until the lower end of the slider *SS* is exactly even with the top of the mercury meniscus as shown. Prove the setting of the slider by raising and lowering the eye slightly to be sure that no light can be

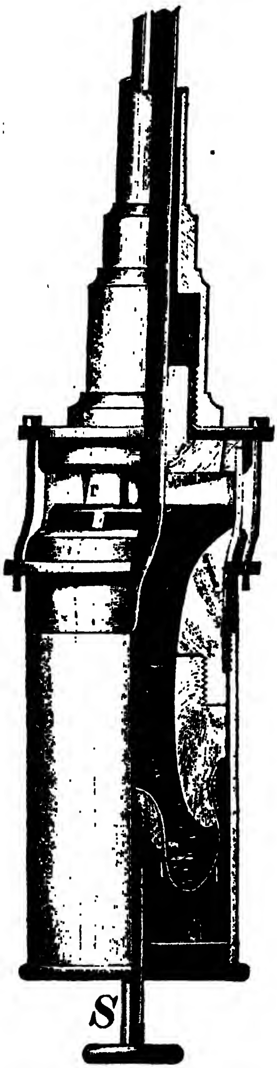


Fig. 12.

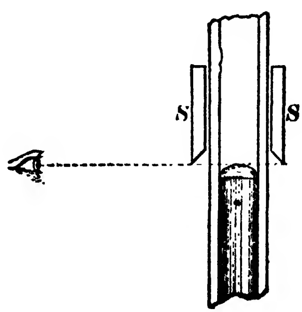


Fig. 13.

seen between the edges S and S and the topmost point of the meniscus. Then take the vernier reading.

The thermometer which is attached to the barometer tube should be read every time a barometer reading is taken.

If it is desired to determine the height of the mercury meniscus (the vertical distance from the line of contact of mercury with glass to the topmost point of the mercury), the slider SS, Fig. 13, may be adjusted until its lower edge is on a level with the line of contact of mercury and glass, and the vernier reading taken. The difference between this vernier reading and the one previously obtained will be the height of the meniscus.

To obtain data for the barometer curve and for determining the mean atmospheric pressure during the given interval of time, barometer readings are to be taken every five minutes for one or more hours. At each point of time the following two sets of readings should be taken: just before the given instant adjust and read the barometer; and read the temperature from the thermometer attached to the barometer tube; repeat these readings immediately after the given instant. The average of these two readings will be taken as the barometer reading at the given instant.

The readings should be tabulated, times in the first column, pairs of actual readings in the second column, averages of the pairs of readings in the third column, readings of thermometer in the fourth column, and readings from which the heights of meniscus are to be determined in the fifth column (the height of meniscus need be determined but once).

Computations and results. — The average of all the readings in the second column in the above table is the average barometer reading during the period of the observations. This average reading is to be corrected for the various systematic errors as follows, using the mean temperature during the time.

*Temperature correction.** — The average barometer reading is

* The order in which the various corrections are made is a matter of indifference, inasmuch as each correction is quite small in value.

to be corrected for thermal expansion, that is, to be reduced to what it would be if the temperature had been 0° C. This correction is most easily made by means of the accompanying table. The student should also calculate this correction directly from the coefficients of thermal expansion of brass and mercury as follows: The brass scale of the barometer, which is assumed to be correct at 0° C. is $(1 + 0.000018t)$ times too long at $t°$ C. Therefore the reading of the barometer is to be multiplied by $(1 + 0.00018t)$ to correct for the expansion of the brass scale. The density of mercury is $(1 - 0.000182t)$ times as small at $t°$ C. as at 0° C., and the height of the mercury column is to be multiplied by $(1 - 0.000182t)$ to correct for the expansion of the mercury.

TABLE.

REDUCTION OF METRIC BAROMETER TO 0° C.—BRASS SCALE CORRECT AT 0° C.

Corrections are in millimeters and they are subtractive.

Readings in millimeters.

Temperatures Centigrade.	680	690	700	710	720	730	740	750	760	770	780
10°	1.11	1.13	1.14	1.16	1.17	1.19	1.21	1.22	1.24	1.26	1.27
11	1.22	1.24	1.26	1.27	1.29	1.31	1.33	1.35	1.36	1.38	1.40
12	1.33	1.35	1.37	1.39	1.41	1.43	1.45	1.47	1.49	1.51	1.53
13	1.44	1.46	1.48	1.50	1.53	1.55	1.57	1.59	1.61	1.63	1.65
14	1.55	1.57	1.60	1.62	1.64	1.67	1.69	1.71	1.73	1.76	1.78
15	1.66	1.69	1.71	1.74	1.76	1.78	1.81	1.83	1.86	1.88	1.91
16	1.77	1.80	1.82	1.85	1.88	1.90	1.93	1.96	1.98	2.01	2.03
17	1.88	1.91	1.94	1.97	1.99	2.02	2.05	2.08	2.10	2.13	2.16
18	1.99	2.02	2.05	2.08	2.11	2.14	2.17	2.20	2.23	2.26	2.29
19	2.10	2.13	2.17	2.20	2.23	2.26	2.29	2.32	2.35	2.38	2.41
20	2.21	2.25	2.28	2.31	2.34	2.38	2.41	2.44	2.47	2.51	2.54
21	2.32	2.36	2.39	2.43	2.46	2.50	2.53	2.56	2.60	2.63	2.67
22	2.43	2.47	2.51	2.54	2.58	2.61	2.65	2.69	2.72	2.76	2.79
23	2.54	2.58	2.62	2.66	2.69	2.73	2.77	2.81	2.84	2.88	2.92
24	2.66	2.69	2.73	2.77	2.81	2.85	2.89	2.93	2.97	3.01	3.05
25	2.77	2.81	2.85	2.89	2.93	2.97	3.01	3.05	3.99	3.13	3.17
26	2.88	2.92	2.96	3.00	3.04	3.09	3.13	3.17	3.21	3.26	3.30
27	2.99	3.03	3.07	3.12	3.16	3.20	3.25	3.29	3.34	3.38	3.42
28	3.10	3.14	3.19	3.23	3.28	3.32	3.37	3.41	3.46	3.51	3.55
29	3.21	3.25	3.30	3.35	3.39	3.44	3.49	3.54	3.58	3.63	3.68
30	3.32	3.36	3.41	3.46	3.51	3.56	3.61	3.66	3.71	3.75	3.80
31	3.43	3.48	3.53	3.58	3.63	3.68	3.73	3.78	3.83	3.88	3.93
32	3.54	3.59	3.64	3.69	3.74	3.79	3.85	3.90	3.95	4.00	4.05
33	3.64	3.70	3.75	3.81	3.86	3.91	3.97	4.02	4.07	4.13	4.18
34	3.75	3.81	3.87	3.92	3.98	4.03	4.09	4.14	4.20	4.25	4.31
35	3.86	3.92	3.98	4.03	4.09	4.15	4.21	4.26	4.32	4.38	4.43

Gravity correction. — The reading of a barometer may have different values for the same value of atmospheric pressure, depending upon the intensity of gravity at the place where the observation is taken. It is therefore necessary in accurate work to take account of variations of the intensity of gravity. This is usually done by finding what the given barometer reading would be at 45° north latitude and at sea level. This reduction is made by multiplying the given reading by $g/980.61$, where 980.61 is the intensity of gravity at 45° north latitude and at sea level, and g is the intensity of gravity at the place where the barometer reading is taken. The intensity of gravity at various places in the United States is given in the accompanying table.

TABLE.

INTENSITY OF GRAVITY AT VARIOUS PLACES.

Locality.	Latitude.	Longitude.	Elevation.	Value of g not Reduced to Sea Level.
Bethlehem, Pa.	40° 36′ 23″	75° 15′ 48″	100 meters.	980.200
Boston, Mass.	42 21 33	71 03 50	22 "	980.382
Chicago, Ill.	41 47 25	87 36 03	182 "	980.264
Cincinnati, O.	39 08 20	84 25 20	245 "	979.990
Cleveland, O.	41 30 22	81 36 38	210 "	980.227
Denver, Col.	39 40 36	104 56 55	1,638 "	979.595
Ithaca, N. Y.	42 27 04	76 29 00	247 "	980.286
Kansas City, Mo.	39 05 50	94 35 21	278 "	979.976
Philadelphia, Pa.	39 57 06	75 11 40	16 "	980.182
San Francisco, Cal.	37 47 00	122 26 00	114 "	979.951
St. Louis, Mo.	38 38 03	90 12 13	154 "	979.987
Terre Haute, Ind.	39 28 42	87 23 49	151 "	980.058
Washington, D. C.	38 53 30	77 01 32	10 "	980.100

Capillary correction. — The highest point of the curved surface of the mercury in a barometer tube is at a lower level than it would be if the surface of the mercury were plane. This discrepancy is called the capillary error. This error is nearly constant in a given barometer (given diameter of barometer tube) if the tube is tapped slightly before each reading so as to bring the meniscus to its normal shape, and a barometer scale is usually adjusted so as to give a reading slightly greater than the actual distance from the ivory point in the cistern to the top of the

curved surface of the mercury in the tube, so as to compensate for the capillary error. In the present instance the value of the capillary error is to be determined using the following table, but not applied as a correction to the barometer reading:

TABLE.

CAPILLARY DEPRESSION OF MERCURY IN A GLASS TUBE.

Additive corrections to barometer reading.

Diameter of Tube.	Height of the Meniscus in mm.							
	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.
4	0.83	1.22	1.54	1.98	2.37	....	....	....
5	0.47	0.65	0.86	1.19	1.45	1.80	....	....
6	0.27	0.41	0.56	0.78	0.98	1.21	1.43	....
7	0.18	0.28	0.40	0.53	0.67	0.82	0.97	1.13
8	....	0.20	0.29	0.38	0.46	0.56	0.65	0.77
9	....	0.15	0.21	0.28	0.33	0.40	0.46	0.52
10	....	....	0.15	0.20	0.25	0.29	0.33	0.37
11	....	....	0.10	0.14	0.18	0.21	0.24	0.27
12	....	....	0.07	0.10	0.13	0.15	0.18	0.19
13	....	....	0.04	0.07	0.10	0.12	0.13	0.14

Simultaneous correction for temperature, gravity, and capillary errors.—These three systematic errors of a barometer are usually quite small, and the order in which the corrections are made is a matter of indifference. The barometer reading may be corrected for all three errors by using the following equation:

$$B = b(1 + 0.0000018t)(1 - 0.000182t)\frac{g}{980.61} + C,$$

in which B is the corrected reading, b is the actual reading, t is the temperature centigrade, g is the value of gravity at the given place, and C is the capillary depression as taken from the above table.

In addition to the determination of the mean barometer reading during the time that the observations were taken and the correction of this mean reading for temperature and gravity, a curve is to be plotted of which the abscissas represent the times in the first column of the table of observations, and the ordinates

represent the averages of the respective pairs of readings as given in the third column of the table of observations. In laying off the ordinates, the first two figures of the barometer readings may be omitted. For example, if the smallest reading is 75.68 centimeters, the ordinates may be plotted to represent the excess of the respective readings above 75 centimeters. This makes it possible to use a scale of ordinates large enough to show the fluctuations of atmospheric pressure distinctly. When the various points have been located in the plot, they should be joined by fine straight lines,* inasmuch as nothing is known concerning the fluctuations of atmospheric pressure in the intervals between the respective pairs of barometer readings.

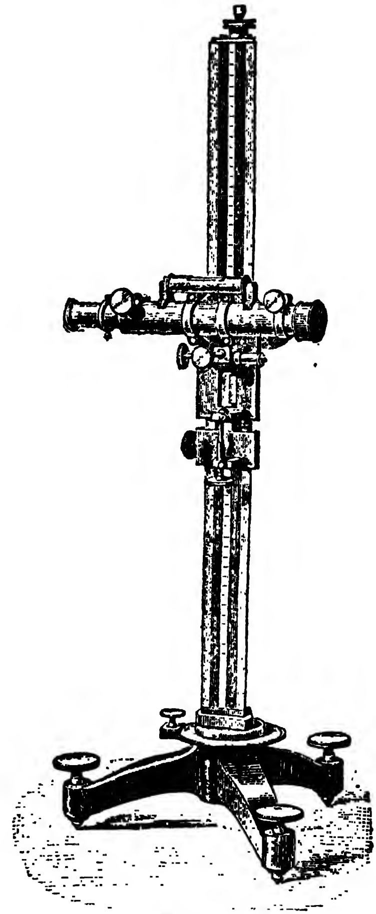


Fig. 14.

EXPERIMENT 4.

The Cathetometer.

The object of this experiment is to afford practice in the use of the cathetometer for the determination of a difference of level.

Apparatus.—Fig. 14 shows a common form of the cathetometer. It consists of a vertical column upon one side of which a scale is etched. A carriage

* Usually a smooth curve is drawn among a number of plotted points. In the present instance, however, it is not desirable to do this, because the actual value of the atmospheric pressure varies by fits and starts; a smooth curve would signify nothing.

slides up and down this column; upon this carriage a horizontal sighting telescope is mounted; and, attached to the carriage, is a vernier, the readings of which indicate the position of the telescope and carriage upon the vertical scale. The difference in level between two marks is determined by sighting the horizontal telescope first upon one mark and then upon the other mark and taking the corresponding readings of the vernier. The difference of these vernier readings is equal to the vertical distance between the two marks, provided the instrument be in adjustment. The necessary adjustments are as follows:

1. The spirit level which is attached to the telescope must be so adjusted as to indicate accurately when the telescope tube is horizontal.

2. The cross-hairs of the telescope must be so adjusted that the line drawn from the intersection of the cross-hairs to the center of the object glass of the telescope lies in the axis of the telescope tube.

3. The column must be adjusted to a vertical position. This

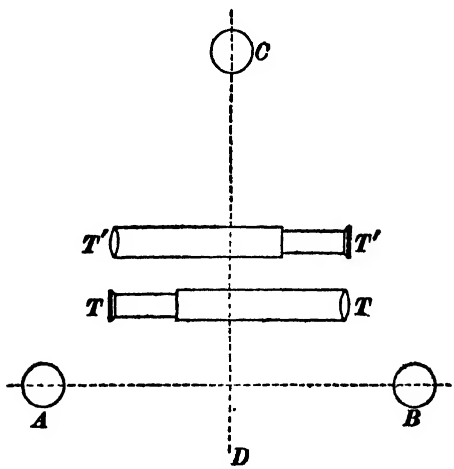


Fig. 15.

adjustment is made as follows: Loosen the clamp at the base of the column and turn the column until the telescope lies in the position *TT*, Fig. 15, parallel to the line *AB* where *A*, *B*,

and C represent the leveling screws of the tripod. Then level the telescope by means of its own leveling screw. Turn the column through 180°, bringing the telescope into the position $T'T'$, again parallel to AB. It will now be found that the telescope is not level. Re-level the telescope, this time making one half of the adjustment by means of the leveling screw of the telescope and one half of the adjustment by means of the leveling screw A or B. Then turn the telescope into position TT and again re-level the telescope, half by its own leveling screw and half by A or B. Proceed in this manner until the telescope is accurately level in both positions TT and $T'T'$. Now turn the column through 90°, bringing the telescope parallel to the line CD, Fig. 15, and level the telescope wholly by adjusting the leveling screw C. The column is now in a vertical position.

4. The telescope must be accurately re-levelled before each reading by adjusting the leveling screw of the telescope.

5. The telescope must be properly focused. This means that the image of the bench-mark (formed by the object glass of the telescope) must lie in the same plane with the cross-hairs. Failure of this condition is indicated by relative motion of cross-hairs and bench-mark when the eye is moved slightly up and down. The proper adjustment may be made by first focusing the cross-hairs very sharply by moving the eye-piece tube, and then focusing the bench mark by turning the milled head at one side of the telescope. It frequently happens that the sliding tube which is moved by the milled head fits loosely into the telescope tube so that a slight rocking motion is given to the sliding tube when the milled head is turned back and forth. Errors due to this fault may be to some extent eliminated by always completing the adjustment of focus by turning the milled head in one particular direction.

Adjustments 1 and 2 will be made before the instrument is placed in the student's hands. Adjustment 3 must be made at the beginning of the experiment, and a slight error in this adjustment will have no appreciable effect upon the results. Adjust-

ments 4 and 5 must be made with greatest care before each reading is taken.

In sighting the telescope upon a bench-mark, proceed as follows: Loosen the screw which clamps the carriage to the vertical column and move the telescope up or down until, by sighting along the telescope tube, it is seen that the axis of the telescope is approximately on a level with the bench-mark. Then clamp the carriage and adjust the slow motion screw until the cross-hairs are exactly coincident with the bench-mark as seen through the telescope.

Caution. — In moving the telescope up and down, do not take hold of the telescope, *take hold of the carriage.* In clamping the carriage in the desired position, a very moderate pressure of the clamping screw is sufficient; excessive pressure is to be avoided.

Work to be done. — (*a*) *Preliminary practice.* — Clamp a meter scale in a vertical position at a distance of about one meter from the cathetometer, and make repeated measurements of a given length of this scale. In making the measurements, it is well to sight upon the upper mark first and then upon the lower mark, in order to avoid a possible disarrangement of the cathetometer by the very considerable force that is necessary to raise the sliding carriage. Repeat this measurement until it is seen that successive values agree closely with each other.

(*b*) *To measure any difference of level.* — Sight upon the upper point first and then upon the lower point. If the points do not lie in a vertical line, the column of the cathetometer must be turned upon its base in bringing the telescope from one position to the other. In this manner measure the difference of level between two bench-marks fixed upon the laboratory wall. This measurement must be repeated several times.

(*c*) *To standardize a barometer scale.* — Hang a barometer at a distance of about one meter from the cathetometer. Tap the barometer scale gently and measure the height of the mercury meniscus by means of the barometer vernier, as explained in experiment 3. The data as to the bore of the barometer tube will

be supplied by the instructor. Using the cathetometer, measure the vertical distance between the 760 millimeter mark and the ivory point *r*. This measurement should be repeated several times.

Computations and results. — From the data taken in (*b*), find the average of the measured values of the difference of level of the two bench-marks, and determine the probable error of this result.

From the data taken in (*c*), find the average of the measured values of the distance between the 760 millimeter mark and the ivory point *r* of the barometer, and determine the probable error of this result. From the table in Experiment 3, find the capillary depression corresponding to the measured height of meniscus and given bore of tube. Subtract this depression from 760 millimeters, and compare the remainder with the measured distance between the 760 mark and ivory point.

EXPERIMENT 5.

Study of a Divided Circle.

The object of this experiment is to study the inaccuracies of a divided circle.

Apparatus. — The apparatus to be used in this experiment is a divided circle with two verniers carried at opposite ends of an alidade. Such a circle is subject to the following inaccuracies:

1. *Inaccuracy in location of the divisions of the circle.* — This inaccuracy may be entirely erratic, the divisions falling promiscuously some to the one side and some to the other side of the positions they should occupy; or it may consist in the crowding together of the divisions in one portion of the circle and the spreading apart of the divisions in another portion of the circle.

2. *Eccentricity of alidade.* — The axis about which the vernier arm, or alidade, revolves may not coincide with the center of the divided circle. This error of centering may cause considerable error in the measured value of an angle if but one vernier is used; whereas by the use of two verniers at approximately 180° apart,

the error due to the eccentricity of the alidade is eliminated. Let C, Fig. 16, be the center of the circle and A the axis of rotation of the alidade. The angle to be measured is the angle α through which the alidade has been turned, carrying one vernier from a to a' and the other from b to b', so that the value of the angle as indicated by the vernier a is the arc aa' (equals the angle aCa') and the value of the angle as indicated by the vernier b is the arc bb' (equals the angle bCb'). It can be shown by geometry that the true value of the angle α through which the alidade has been turned is equal to one half the sum of the angles aCa' and bCb'. Therefore by averaging the readings of the two verniers, the error due to eccentricity of the alidade is eliminated.

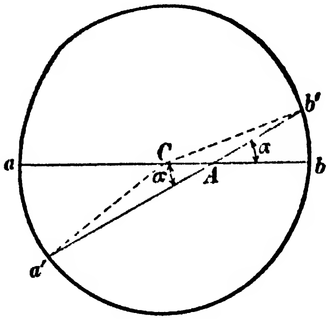


Fig. 16.

3. *Inaccuracy of setting of verniers.* — The verniers are supposed to lie with their zero points on a straight line passing through the axis of rotation of the alidade. It frequently happens, however, that this condition is not realized. This inaccuracy will be referred to as a *bent alidade*, for brevity. The effect of a bent alidade upon the measured value of an angle is always negligible; but this inaccuracy is important in the study of the eccentricity errors of the circle. The readings of the two verniers on a divided circle should differ by 180°. Eccentricity of the alidade causes the two vernier readings to differ by more or less than 180°, and a bent alidade also causes the two vernier readings to differ by more or less than 180°. The effects of eccentricity error and the effect of bent alidade may be separated, however, because the former shows itself as a *variable difference* between vernier readings, and the latter shows itself as a *constant difference* between vernier readings. Thus, Fig. 17 shows the two verniers a and b rotating about the true center of the

circle C in which case the difference between the vernier readings is ϕ° less than 180°; and Fig. 18 shows the two verniers a and b rotating about the point A. In this case the vernier readings a and b are exactly 180° apart, whereas the vernier

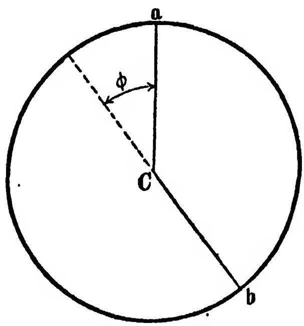


Fig. 17.

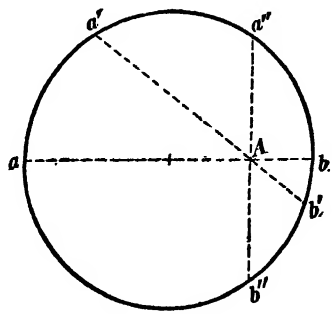


Fig. 18.

readings a' and b' and a'' and b'' are very far from being 180° apart.

Work to be done. — Set vernier a accurately at 0°, 10°, 20°, and so on throughout the whole circle, and read the vernier b for each position of vernier a. Repeat these readings and average the two readings of vernier b thus obtained for each position of vernier a.

Computations and results. — Subtract the readings of vernier a from the corresponding readings of vernier b, and deduct 180° from the remainders. The final differences d so found depend partly upon the eccentricity of the alidade and partly upon the bend in the alidade, and these two effects may be separated as follows: The average of all the values of d above found, due regard being had for algebraic signs, gives the value of the angle ϕ in Fig. 17 due to bend of alidade. This value of ϕ may then be subtracted from the values of d, thus giving the true eccentricity errors, affected more or less by errors of reading and local errors of the circle. Plot these values of $(d - \phi)$ as ordinates and the corresponding readings of the a vernier as abscissas and draw through the plotted points a smooth sine curve which passes

nearest to all the plotted points. The ordinates of this sine curve will then represent the true values of the eccentricity errors.

Example. — The following table gives the results of a test of a

Readings of Vernier a.	Values of d.		$d-\phi$.	Readings of Vernier a.	Values of d.		$d-\phi$.
	Negative.	Positive.			Negative.	Positive.	
60°	....	0.5′	0.02′	240°	....	0.0′	−0.52′
70	0.0′	....	0.52	250	....	2.0	+1.47
80	0.5	....	1.02	260	....	1.0	+0.47
90	0.0	....	−0.52	270	....	2.0	+1.47
100	0.0	....	−0.52	280	....	2.0	+1.47
110	1.0	....	−1.52	290	....	2.0	+1.47
120	2.0	....	−2.52	300	....	1.0	+0.47
130	1.0	....	−1.52	310	....	2.0	+0.47
140	1.0	....	−1.52	320	....	2.0	+1.47
150	2.0	....	2.52	330	....	1.5	+0.97
160	2.0	....	−2.52	340	....	2.0	+1.47
170	2.0	....	2.52	350	....	2.5	+1.97
180	0.0	....	−0.52	360	....	2.0	+1.47
190	1.0	....	−1.52	10	....	1.0	+0.47
200	0.0	....	−0.52	20	....	1.5	+0.97
210	0.0	....	−0.52	30	....	2.0	+1.47
220	....	1.0′	0.47	40	....	2.0	+1.47
230	....	0.5	−0.02	50	....	1.0	+0.47

divided circle in the Sloane Physical Laboratory of Yale University. The abscissas of the small circles in Fig. 19 represent the readings of vernier a, and the ordinates of the small circles rep-

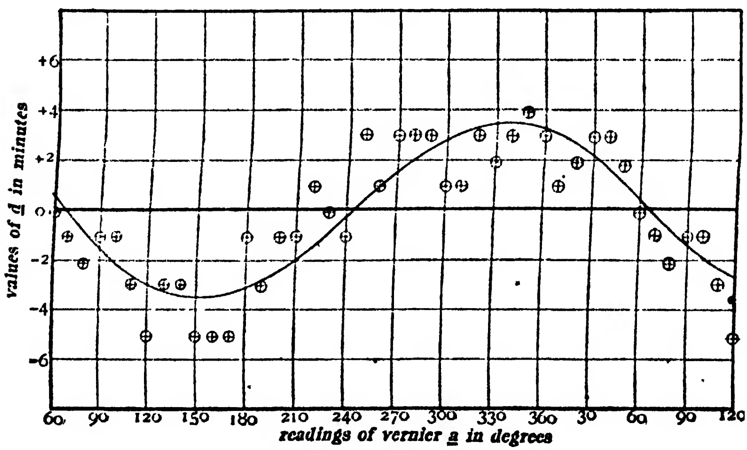


Fig. 19.

resent the values of $d - \phi$ as explained above. The ordinates of the sine curve in Fig. 19 represent, as nearly as one can approximate to them from the given observations, the values of the true eccentricity errors of the circle. The eccentricity errors are zero at 66° and 246°; that is to say, the axis of rotation of the alidade in the given circle lies on a line passing through the 66° and 246° divisions. The extremely irregular arrangement of the plotted points in Fig. 19 is due largely to the fact that the vernier readings could be taken only to single minutes.

EXPERIMENT 6.

The Reflecting Goniometer.

The object of this experiment is to afford practice in the use of the goniometer by measuring the angles between the faces of a prism. The instrument here used, however, is an ordinary

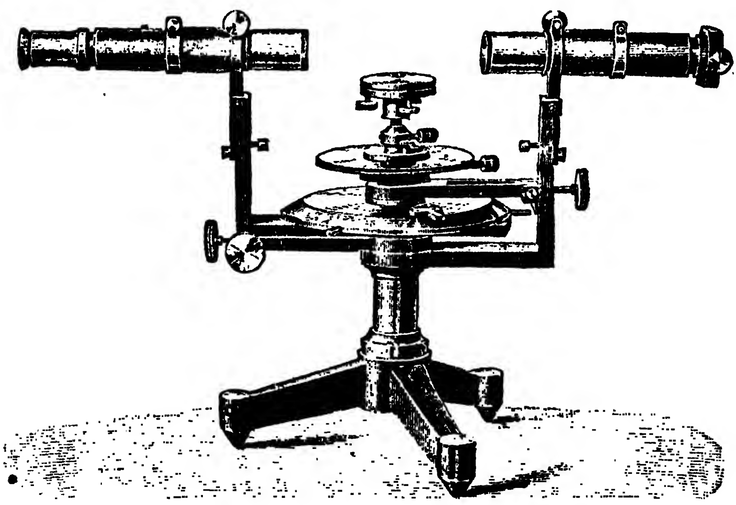


Fig. 20.

spectrometer, a general view of which is shown in Fig. 20, and a sketch of the top of which is shown in Fig. 21.

Apparatus. — The instrument as shown in Fig. 21 is arranged for the measurement of the angle between two faces of a prism. Light from a lamp passes through a narrow slit S which is at the principal focus of a lens L. After being reflected from the face of the prism, the light enters the telescope T, and an image of the slit S is formed in the focal plane p of the telescope. The small circular table upon which the prism is mounted is then turned until the image of S due to reflection from a second face of the prism is in the same position as before. Then the angle turned by the prism table as indicated by the divided circle of the instrument is equal to the angle between the normals NN, Fig. 21.

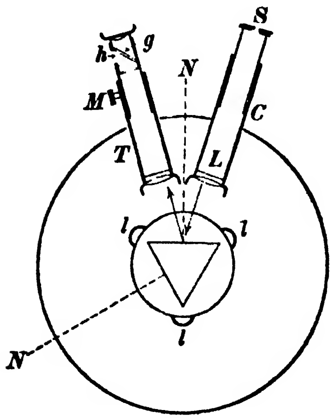


Fig. 21.

Before undertaking to use the spectrometer either for making adjustments or for making measurements, it is quite important that the student familiarize himself with the important details of the instrument, finding out especially which clamping screws control the motion of the arms that carry the collimator C and the telescope T, which screws control the motion of the prism table and of the divided circle, and which screws are to be used for adjusting the tubes of collimator and telescope into the desired direction. For this experiment the clamp screws are to be set so that the telescope, collimator and verniers remain stationary, while the divided circle and prism turn together. A slow motion screw is provided for final adjustment. In using the instrument always move the telescope and collimator by taking hold of the arms, not by taking hold of the tubes, and avoid excessive pressure on the clamping screws.

The collimating eye-piece. — The sighting telescope is usually provided with what is called a collimating eye-piece, the ar-

rangement and action of which are shown in Fig. 22. Light from a lamp enters the hole h at one side of the eye-piece and strikes a diagonal piece of clear glass g from which a portion of the light is reflected towards the object glass of the telescope passing by the cross-hairs c and thus making the cross-hairs luminous. After passing through the telescope objective the

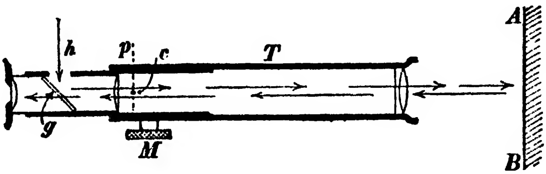


Fig. 22.

light strikes a plane surface AB which reflects the light directly back into the telescope with the result that the object glass forms an image of the cross-hairs along side of the actual cross-hairs. If the surface AB is exactly perpendicular to the axis of the telescope this image of the cross-hairs will be coincident with the cross-hairs themselves.

(*a*) *Preliminary approximate adjustment.* — Set the collimator and telescope tubes so that their axes are parallel to the plane of the circle and point toward the axis of rotation of the circle, as nearly as can be judged by the eye. Fasten the prism to the circular platform or table either by means of the clamp provided for the purpose or with a little wax, being careful to have the face of the prism as nearly as possible perpendicular to the plane of the circle. The prism should be so placed that its corners will be directly over the three leveling screws lll under the small circular platform, as shown in Fig. 21.

(*b*) *To focus the telescope for parallel rays.* — It is desirable, even when the telescope has a collimating eye-piece, to focus the telescope for parallel rays by sighting the telescope at a distant object through an open window. First adjust the sliding tube at the extreme end of the telescope eye-piece to bring the cross-hairs sharply into focus, and then turn the milled head M, Fig. 21,

until the distant object is sharply in focus. This adjustment is to be left unchanged throughout the entire experiment.

If the telescope has a collimating eye-piece, the focus for parallel rays should be tested as follows in order that the student may become familiar with the use of the collimating eye-piece before undertaking adjustment (*c*). Place the lamp at one side of the telescope so that the light from the lamp may enter the aperture *h* in the collimating eye-piece. Turn the prism table until one of the prism faces is as nearly as possible perpendicular to the axis of the telescope, and adjust the position of the lamp until the field of the telescope is seen to be illuminated.* Then focus the telescope until the reflected image of the cross-hairs is seen along side of the cross-hairs themselves, adjust the focus until the cross-hairs and their image are both as distinct as possible.

(*c*) *To set the face of the prism parallel to the axis of rotation and perpendicular to the axis of the telescope.*† — With light shining through the aperture of the collimating eye-piece, and the prism so located as to show the reflected image of the cross-hairs, adjust the direction of the telescope tube until the reflected image of the cross-hairs coincides with the cross-hairs themselves, turning the prism table slightly to bring about a horizontal adjustment of the position of the reflected image. Then turn another face of the prism into position and bring the reflected image of the cross-hairs into coincidence with the cross-hairs themselves, *half* by adjusting the axis of the telescope, and *half* by adjusting the level of the prism table, making a horizontal adjustment by turning the prism table as before. In adjusting the level of the prism table, use the leveling screw under the corner of the prism which is opposite to the reflecting face. Proceed in this manner, turning the faces of the prism successively into position, and repeating this

* This may require three adjustments to be made simultaneously, turning of prism table, changing of inclination of the telescope tube, and moving of the lamp; and if the telescope has not been previously focused, at least approximately, for parallel rays a fourth adjustment is involved before the cross-hairs can be seen.

† This adjustment cannot be easily made if the telescope is not provided with a collimating eye-piece.

half-and-half adjustment, until it is found that no further adjustment is needed.

(*d*) *To focus the collimator.* — With the collimator and telescope set as shown in Fig. 21, place a source of light before the slit *S*, turn the prism so that the light from the collimator is reflected into the telescope, and adjust the sliding tube of the collimator until the image of the slit is seen to be sharply focused in the telescope. The collimator is then in focus for parallel rays.

Work to be done. — The angles of a prism are to be measured by each of the following methods:

Method 1. Make adjustments *a*, *b*, and *c*, as described above. Then, leaving the telescope and the lamp undisturbed, turn the circle and prism table together (verniers and telescope stationary), until the reflected image of the cross-hairs produced by a given face *A* of the prism is coincident with the cross-hairs themselves, and read the verniers. Then turn the prism and circle until the reflected image of the cross-hairs from face *B* is coincident with the cross-hairs themselves, and read the verniers. Then turn the prism and circle until the reflected image from face *C* of the prism is coincident with the cross-hairs themselves, and read the verniers. This operation should be repeated several times.

Method 2. Place a source of light in front of the slit of the collimator, and make adjustment *d*. Then turn the circle and prism until the reflected image of the slit from face *A* of the prism is coincident with the point of intersection of the cross-hairs and read the verniers. Turn the circle and prism until the reflected image of the slit from face *B* of the prism is coincident with the point of intersection of the cross-hairs, and read the verniers. Then turn the circle until the reflected image of the slit from face *C* is coincident with the point of intersection of the cross-hairs, and read the verniers. These readings should be repeated several times.

Computations and results. — Find the mean of the readings of each vernier for each position of the prism and circle for method

1 and for method 2. The differences of these vernier readings are the values of the angle between the normals to the respective surfaces of the prism. From these values compute the three angles of the prism.

EXPERIMENT 7.

THE MICROMETER CALIPER.

The object of this experiment is to determine the mean diameter, cross-sectional area, and the gauge number of each of several samples of wire, and to determine the mean diameter and volume of several bicycle balls.

Apparatus. — *The micrometer caliper is shown in Fig. 4.* When the screw is properly adjusted the distance between the caliper tips is indicated directly by the reading of the screw. Whole turns are indicated by the fixed scale which is parallel to the screw, and fractions of a turn are indicated by the scale on the rotating drum. Before using the instrument one must know the pitch of the screw * in fractional parts of an inch or centimeter, and one must know the number of divisions on the rotating drum.

The reading of the caliper when the tips are brought into contact is equal to zero when the caliper is in proper adjustment. It is usually desirable, however, to observe the reading of the screw when the caliper tips are in contact, and apply this reading as a correction to subsequent readings of the instrument.

In using the micrometer caliper, never jamb the screw. A certain slight pressure upon the object which is being calipered is sufficient.

Work to be done. — (*a*) Determine the reading of the screw when the caliper tips are in contact, the zero reading, as it is called.

* The pitch of the screw is always a simple fractional part of an inch or centimeter and it may be determined by measuring the scale of whole turns by means of a scale of inches or centimeters.

(*b*) Measure three mutually perpendicular diameters of each of the bicycle balls.

(*c*) Measure two mutually perpendicular diameters near each end and near the middle of each of several pieces of wire.

TABLE GIVING THE DIAMETERS IN DECIMAL PARTS OF AN INCH OF WIRES CORRESPONDING TO THE NUMBERS OF VARIOUS GAUGES.

No. of Wire.	Brown & Sharp.	Birmingham, or Stubbs'.	Washburn & Moen Mfg. Co.	Trenton Iron Co.	G. W. Prentiss.	Old English.
0000	0.46	0.454	0.393	0.4	....	....
000	.40964	.425	.362	.36	0.3586	....
00	.3648	.38	.331	.33	.3282	....
0	.32486	.34	.307	.305	.2994	....
1	.2893	.3	.283	.285	.2777	....
2	.25763	.284	.263	.265	.2591	....
3	.22942	.259	.244	.245	.2401	....
4	.20431	.238	.225	.225	.223	....
5	.18194	.22	.207	.205	.2047	....
6	.16202	.203	.192	.19	.1885	....
7	.14428	.18	.177	.175	.1758	....
8	.12849	.165	.162	.16	.1605	....
9	.11443	.148	.148	.145	.1471	....
10	.10189	.134	.135	.13	.1351	....
11	.090742	.12	.12	.1175	.1205	....
12	.080808	.109	.105	.105	.1065	....
13	.071961	.095	.092	.0925	.0928	....
14	.064084	.083	.08	.08	.0816	0.083
15	.057068	.072	.072	.07	.0726	.072
16	.05082	.065	.063	.061	.0627	.065
17	.045257	.058	.054	.0525	.0546	.058
18	.040303	.049	.047	.045	.0478	.049
19	.03589	.042	.041	.04	.0411	.04
20	.031961	.035	.035	.035	.0351	.035
21	.028062	.032	.032	.031	.0321	.0315
22	.025307	.028	.028	.028	.029	.0295
23	.022571	.025	.025	.025	.0261	.027
24	.0201	.022	.023	.0225	.0231	.025
25	.0179	.02	.02	.02	.0212	.023
26	.01594	.018	.018	.018	.0194	.0205
27	.014195	.016	.017	.017	.0182	.01875
28	.012641	.014	.016	.016	.017	.0165
29	.011257	.013	.015	.015	.0163	.0155
30	.010025	.012	.014	.014	.0156	.01375
31	.008928	.01	.0135	.013	.0146	.01225
32	.00795	.009	.013	.012	.0136	.01125
33	.00708	.008	.011	.011	.013	.01025
34	.006304	.007	.01	.01	.0118	.0095
35	.005614	.005	.0095	.0095	.0109	.009
36	.005	.004	.009	.009	.01	.0075
37	.004453	....	.0085	.0085	.0095	.0065
38	.003965	....	.008	.008	.009	.00575
39	.003531	....	.0075	.0075	.0083	.005
40	.003144	....	.007	.007	.0078	.0045

No. of Wire.	Twist Drill Gauge.	No. of Wire.	Twist Drill Gauge.	No. of Wire.	Music Wire Gauge.	Zinc Gauge.
1	0.2280	31	0.1200	1	....	....
2	.2210	32	.1160	2	0.0105	....
3	.2130	33	.1130	3	.0115	....
4	.2090	34	.1110	4	.0125	....
5	.2055	35	.1100	5	.0145	....
6	.2040	36	.1065	6	.015	....
7	.2010	37	.1040	7	.0175	....
8	.1990	38	.1015	8	.019	0.016
9	.1960	39	.0995	9	.022	.018
10	.1935	40	.0980	10	.0245	.020
11	.1910	41	.0960	11	.027	.024
12	.1890	42	.0935	12	.0285	.028
13	.1850	43	.0890	13	.0305	.032
14	.1820	44	.0860	14	.032	.036
15	.1800	45	.0820	15	.035	.040
16	.1770	46	.0810	16	.036	.045
17	.1730	47	.0785	17	.038	.050
18	.1695	48	.0760	18	.040	.055
19	.1660	49	.0730	19	.042	.060
20	.1610	50	.0700	20	.043	.070
21	.1590	51	.0670	21	.0445	....
22	.1570	52	.0635	22	.047	....
23	.1540	53	.0595	23	.049	....
24	.1520	54	.0550	24	.053	....
25	.1495	55	.0520	25	.056	....
26	.1470	56	.0465	26	.0605	....
27	.1440	57	.0430	27	.064	....
28	.1405	58	.0420	28	.0685	....
29	.1360	59	.0410	29	.0715	....
30	.1285	60	.0400	30	.076	....

Computations and results. — Calculate the volume of each of the bicycle balls. Calculate the sectional area of each sample of wire in square inches or square centimeters and in circular mils. From the accompanying tables find the gauge number of each sample of wire.

Wire gauges. — Size of wire and thickness of sheet metal are generally specified in commerce by trade numbers, different schemes of numbering being employed by different manufacturers. Manufacturers of copper and german silver wire in America use mainly Brown & Sharpe's gauge (B. & S.); the thickness of sheet copper, brass, and german silver is also generally specified by Brown & Sharpe's gauge numbers. Iron wire is usually numbered by what is called the Trenton gauge. Brass wire is generally numbered by the Old English gauge. Piano

steel wire is usually numbered by the music wire gauge and tool steel wire is usually numbered by Stubb's gauge.

EXPERIMENT 8.

THE SPHEROMETER.

The object of this experiment is to determine the radius of curvature of the surface of a lens by means of the spherometer.

Apparatus. — The essential features of the spherometer are shown in Fig. 23. The instrument is used mainly for determining the curvature of the surfaces of lenses. The micrometer screw, which is held in a vertical position in the center of a tripod, has a large circular head which is usually divided into 500 parts for indicating fractions of a turn. Attached to one corner of the tripod is a vertical scale the reading upon which of the edge of the circular head indicates whole turns of the screw, and the reading of the edge of the vertical scale on the divided head indicates fractions of a turn.

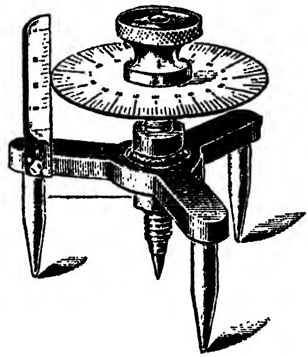


Fig. 23.

Work to be done. — (*a*) Determine the pitch of the screw as described in Experiment 7 on the micrometer caliper, and make a few readings with the instrument for practice.

(*b*) *To determine the thickness of a piece of plate glass.* — Place the spherometer on a true plane, turn the circular head until the tip of the micrometer screw is barely in contact with the plane, and take the reading of the instrument. In bringing the screw into contact with the true plane proceed as follows: Turn the screw downwards carefully until the tip of the screw is evidently in contact with the plane; then give to the instrument a slight rocking motion, and at the same time turn the screw upwards

very slowly until the chattering noise due to the rocking of the instrument can no longer be perceived; then take the reading. In repeating observations, do not watch the divisions of the circular head while setting the screw.

Next, place the piece of glass on the true plane, turn the tip of the screw into contact with this piece of glass, and take the readings of the instrument as before. Turn the piece of glass over and repeat. If the glass is slightly concave on one side, the average of the spherometer readings on one side will be found to be greater than the average of the readings of the other side. *The greater value is in error on account of the curvature of the piece of glass, and the lesser of the two values should be used in* computing the thickness of the piece of glass.

(*c*) *To determine the radius of curvature of the surface of a lens.* — Take the reading of the instrument on a true plane as before, then place the instrument upon the surface of the lens and take the reading. Then measure the distance, from the tip of the screw to each of the tripod points, the screw point having first been adjusted to the true plane.

Computations and results. — (*a*) Determine the thickness of a piece of glass from the observations (*b*), above, and determine the probable error of the result.

Determine the radius of curvature of the surface of the lens from the formula

$$R = \frac{x^2}{2h} + \frac{h}{2}$$

in which x is the average distance of the three tripod points from the tip of the micrometer screw, h is the difference of the readings of the spherometer on the plane and on the lens, and R is the radius of the sphere of which the surface of the lens is a portion.

EXPERIMENT 9.

The Micrometer Microscope.

The object of this experiment is to afford an exercise in the use of the micrometer microscope in the measurement of the bore of a capillary tube and in the determination of the distance between fine rulings on a glass plate.

Apparatus. — Micrometer microscopes are of two kinds, namely, (*a*) the kind in which the entire microscope is moved on a sliding carriage by means of a micrometer screw, and (*b*) the kind in which the cross-hairs in the microscope eye-piece are moved sidewise by means of a micrometer screw. Both kinds of micrometer microscope are shown attached to a comparator in Fig. 25. Figure 24 shows the micrometer microscope of the second kind which is to be used in this experiment.

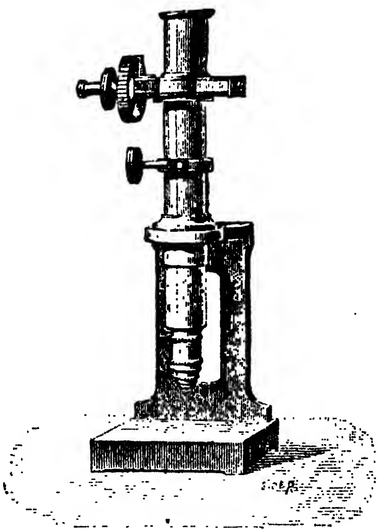


Fig. 24.

In using the micrometer microscope great care must be exercised to avoid turning the screw too far and thus damaging the small metal frame which carries the movable cross-hairs. The eye-piece should be adjusted to bring the cross-hairs into focus before the instrument is focused upon the object to be measured.

Work to be done. — (*a*) *To determine the constant of the instrument.* — Place a standard scale on the stage of the microscope, focus the instrument sharply, turn the screw until the movable cross-hairs are in coincidence with a chosen mark on the scale, and read the instrument. Then turn the screw until the mov-

able cross-hairs have traversed a known number of divisions of the standard scale and again read the instrument.

In reading the instrument the turns of the screw are indicated by the notches at one side of the field of view, as seen in the microscope, and fractions of a turn are indicated by divisions of the circular head of the micrometer screw.

The constant of the screw is the number of turns corresponding to a known unit of length.

(*b*) *To measure the bore of a capillary tube.* Stand a short section of the tube on end so that the end of the bore may be viewed through the microscope, and measure the diameter of the bore in three directions making approximately 60° with each other. It is usually more convenient to turn the microscope in its supporting tube rather than to turn the short piece of glass tube.

(*c*) *To measure the distance between the rulings on a grating.* A grating is a piece of glass or metal with fine lines ruled upon it. Place the grating upon the stage of the microscope with its rulings perpendicular to the axis of the micrometer screw, focus the microscope on the rulings, set the movable cross-hair upon a ruling at one side of the field, and take the reading of the micrometer. Then turn the screw slowly, counting the grating spaces that are passed over until a convenient ruling has been reached near the other side of the field, set the movable cross-hair on this ruling, and again read the micrometer. Record the number of spaces passed over in the above process.

Results. — Average readings of a kind and compute the constant of the microscope in millimeters per turn. Then compute the average bore of the tube and the average grating space in millimeters.

EXPERIMENT 10.

THE USE OF THE COMPARATOR. STANDARDIZATION OF A SCALE.

The object of this experiment is to determine the true value of the distance between two marks on a meter scale by means of the comparator and a standard scale.

Apparatus.—The problem of standardizing a scale is essentially that of measuring an unknown length in terms of a known standard of length. This may be done in either of two ways: (1) By measuring the difference between the unknown length and the length of the standard, or (2) By determining the ratio of the unknown length to the length of the standard. The first and simpler of these two methods is accomplished by means of the comparator, the second is accomplished by means of the dividing engine (see Experiment 11).

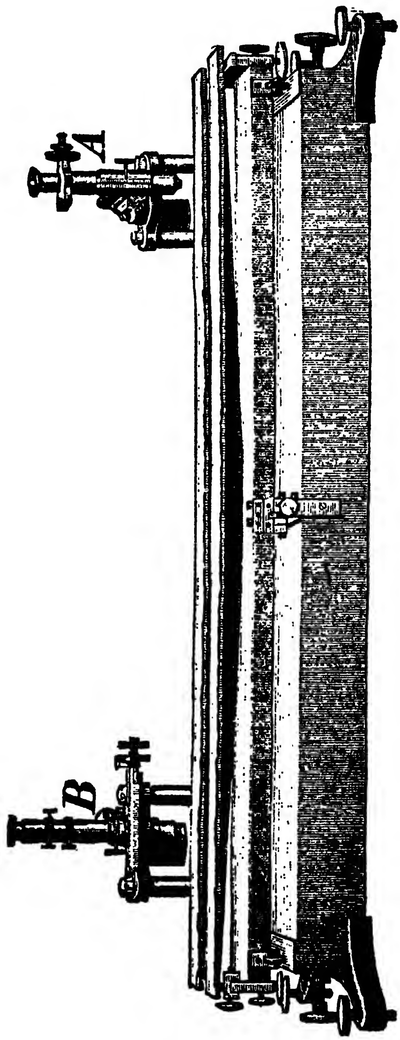


Fig. 25.

A general view of the comparator is shown in Fig. 25. It consists of a heavy metal bed-plate upon which is a carriage supporting two long narrow beams or tables upon which the standard scale and the scale to be standardized are placed. Overhanging the two long beams or tables are two micrometer microscopes. The tables are capable of being adjusted up and down and lengthwise, in order to bring any desired point on one of the scales into sharp focus in the micrometer microscopes. Also the entire carriage with the two long tables can be shifted sidewise so as to bring the one or the other of the scales into position under the microscopes.

It is more convenient and more accurate, in using a scale for measuring length, to use divisions which are marked upon the face of the scale than it is to make use of the extreme ends of the scale. Therefore in the following discussion we shall assume that the length between the 1 centimeter mark and the 99 centimeter mark of the unknown scale is to be standardized in terms of the like length upon the standard scale, and the problem which is presented in the use of the comparator may be understood by referring to Fig. 26. The 1 centimeter marks on the

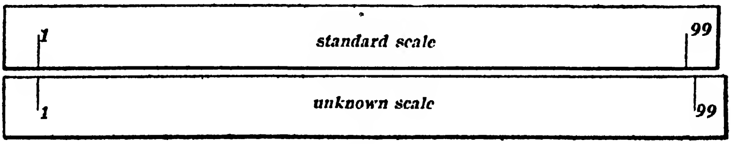


Fig. 26.

two scales are to be brought into exact coincidence, and the distance between the 99 centimeter marks is to be measured.

Work to be done. — Place the standard scale upon one of the long tables of the comparator, and the unknown scale upon the other. Adjust the scales lengthwise by hand until the one centimeter marks come as nearly as possible in line with one of the reading microscopes. Then move the other reading microscope until the 99 centimeter marks are approximately coincident with its axis. Bring first the one and then the other scale under the microscopes by turning the milled head at the end of the comparator (*avoid bumping the carriage against stops*), and adjust the tables up and down and lengthwise by means of the slow motion screws until both microscopes are sharply in focus, and the 1 centimeter marks are exactly coincident with the cross-hair in one of the micrometer microscopes *A*. Bring the standard scale under the microscopes, set the cross-hair of microscope *B* upon the 99 centimeter mark and take the reading of the microscope. Then bring the unknown scale under the microscopes, set the cross-hair of microscope *B* into coincidence with its 99 centimeter mark and take the micrometer reading. In each case the

coincidence of the cross-hair of microscope A with the one centimeter mark should be verified before setting and reading microscope B, and any error of coincidence should be adjusted by moving the long tables. The difference of the two readings of the B microscope is the difference between the length of the standard and the length of the unknown scale expressed in turns of the micrometer screw, and the observer must be careful to note whether the standard scale is longer or shorter than the unknown scale.

To determine the constant of the micrometer B bring the standard scale into its field and observe the number of turns of the micrometer screw required to carry the cross-hair over a chosen number of divisions on the scale.

During the progress of this work the temperature of the scales must be noted at intervals by reading a thermometer placed near them.

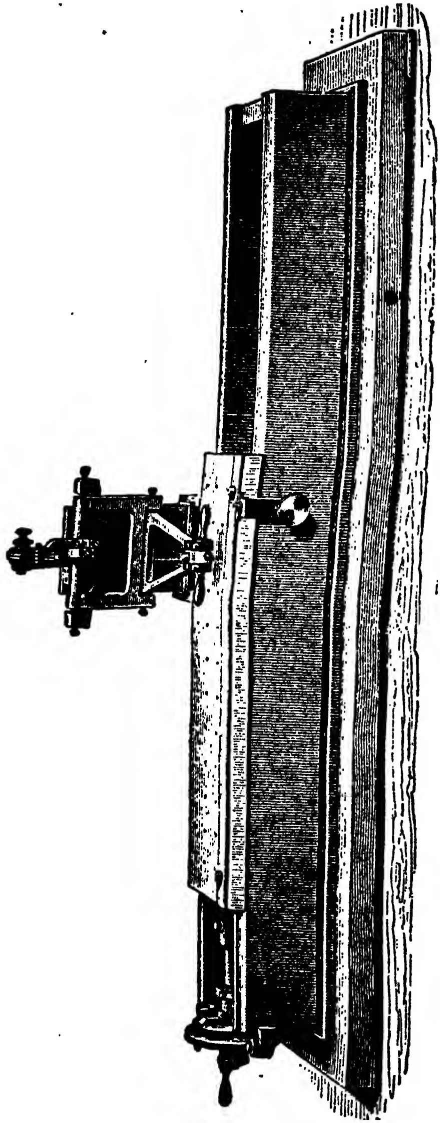
Cautions. — In working with a comparator care should be taken to avoid changes of temperature. Thus, the hands should not be placed upon the instrument more than is absolutely necessary to adjust the various screws and to put the scales in place. Move the carriage only by means of the milled head at the end of the bed plate and especially avoid bumping against the stops. Be careful not to touch the microscope supports inasmuch as a very slight pressure will produce enough flexure to make perceptible errors in the readings. In setting a micrometer microscope always bring the cross-hair to the desired position by turning the screw in a particular direction.

Computations and results. — Reduce the difference in length between the standard and the unknown scale to centimeters or inches as the case may be.

If it is known that the length on the standard scale is 98 centimeters at a given temperature $t°$ C., then its length at the prevailing temperature $t'°$ C. is given by the formula

$$L_{t'} = \frac{98(1 + at')}{(1 + at)}$$

in which a is the coefficient of linear expansion of the standard scale.



In finding the true length of the unknown scale at the temperature t' at which the measurements were taken, the true length L_t' of the standard must be used.

EXPERIMENT 11.

The Dividing Engine.

The object of this experiment is to afford an exercise in the use of the dividing engine.*

Apparatus. — A general view of a dividing engine is shown in Fig. 27. An accurately cut horizontal screw, having a divided circular head for indicating fractions of a turn, engages a nut which pushes a sliding carriage along a track on a heavy metal bed-plate. When the dividing engine is used for measuring lengths a microscope is fixed to the bed-plate and overhangs

* The third exercise, (*c*) is the one that will be generally performed by the student.

the moving carriage. The dividing engine shown in Fig. 27 is equipped only with a graving tool for the making of divided scales.

Work to be done.—(*a*) *To determine the constant of the screw.* Place a standard scale upon the sliding carriage or table, adjust the microscope until the face of the scale is sharply in focus, and adjust the scale until it is accurately parallel to the axis of the screw. In doing this the microscope support must be unclamped and moved from one end of the scale to the other repeatedly, and the two ends of the scale adjusted in succession.

When this adjustment has been made, sight the microscope upon a division at one end of the scale and take the reading of the circular head of the dividing engine screw; then turn the screw, counting whole turns, until the microscope sights at a division at the other end of the standard scale, and again read the circular head. The difference of the two readings is the fraction of a turn which added to the counted turns gives the number of turns of the screw that is required to move the table through the known length on the scale, and from this the constant of the screw may be determined. For example, the screw of a dividing engine is standardized by using 50 centimeters of a standard scale. When the microscope is sighted upon the zero mark of the scale, the circular head reads 0.246 (the head being divided into 1000 parts); in moving the carriage so as to bring the 50 centimeter mark into the axis of the microscope, 124 turns of the screw are required and the micrometer is found to read 0.785; so that 124.539 turns of the screw are equivalent to 50 centimeters. That is, the pitch of the screw is 0.40148 centimeter.

(*b*) *To measure an unknown length.*—Let it be required to standardize the length of a short scale. Place the scale upon the table of the dividing engine and follow the exact procedure described under (*a*), finding the number of turns of the screw which are equivalent to the length of the scale; whence, the screw having been standardized, the true length of the scale is known.

(*c*) *To manufacture a divided scale.* — Knowing the constant of the screw, the number of turns or fraction of a turn equivalent to the divisions into which the scale is to be divided may be calculated. The circular head of the dividing engine is provided with a ratchet device which operates between adjustable stops, and by means of which the circular head may be repeatedly turned through any number of turns or any fraction of a turn up to five or six whole turns. Adjust the stops of this ratchet device so as to allow the ratchet to turn the circular head through the range required for the desired length of divisions of the scale. Then, by moving the ratchet backwards and forwards between the stops, the table on the dividing engine is advanced one scale division for each stroke of the ratchet, care being taken to avoid bumps against the stops. The ruling device on the dividing engine is generally provided with an automatic attachment for making every fifth or every tenth line longer than the others.

Rule a scale 20 centimeters long to fifths of centimeters, and rule a vernier of 20 parts to match this scale. For this purpose metal blanks will be provided. These blanks are to be carefully and uniformly coated with a thin layer of wax and after the divisions have been ruled through the wax, they are to be etched by brushing acid over the wax.*

EXPERIMENT 12.

BASE LINE MEASUREMENT.

The object of this experiment is to measure a distance greatly exceeding the length of the scales which are used. The simplest method is to use two scales, setting them end to end with great care until the length is stepped off. This method is subject to considerable error, owing to the difficulty of standardizing the

* The process of etching is troublesome and requires considerable skill. It may, therefore, be found more suitable to equip the dividing engine with a ruling pen, and substitute blanks made of Bristol board for the metallic blanks. The edges of these cardboard blanks may be neatly trimmed by means of a straight edge and a sharp knife after the rulings have been made.

ends of the scale. For accurate work it is better to leave the ends of the scales a short distance apart in each position, the distance between chosen marks on the two scales being measured by means of a small auxiliary scale as described below.

There are three sources of error in making such measurements, as follows:

1. *Inaccuracy of scales used.* — This error is to be eliminated by standardizing the scales as explained in Experiment 10.

2. *Thermal expansion.* — If the scales used are standardized at a temperature of 0° C. they will be $(1 + at)$ times as long at $t°$ C., a being the coefficient of linear expansion of the material of which the scales are made. This source of error is allowed for by multiplying the measured value of the length by $(1 + at)$, where t is the prevailing temperature during the progress of the measurement. If the scales used are standardized at $t'°$ C., the measured value of the distance must be multiplied by $(1 + at)/(1 + at')$.

3. *Errors of observation*, including errors of setting scales, errors of estimating fractions, and the like. These errors cannot be eliminated, but they can be greatly reduced by careful manipulation and observation and by repeating the measurement a number of times so as to make the probable error of the result small. When the end-to-end method is employed, there is slightly more liability to error due to the danger of displacing one scale when the other is brought into contact with it, and on account of the fact that the end division on a cheap scale is nearly always in error.

Work to be done. — (*a*) Standardize the scales to be used. This can best be done by means of the comparator (see experiment 10).

(*b*) Measure the given distance from left to right and from right to left alternately several times, as follows: Place the two meter scales as shown in Fig. 28, scale A overlapping the starting point L, and a gap of two or three millimeters intervening between the two scales A and B. A small auxiliary scale is

placed as shown for the purpose of measuring the distance between chosen lines, say mark *99* on the one scale and mark *1* on the other. Scale *A* is then placed beyond scale *B*, and

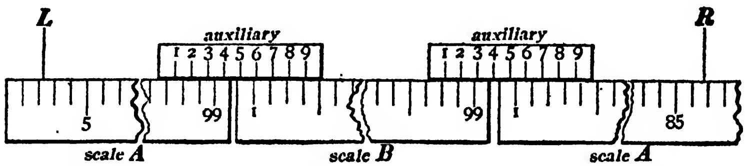


Fig. 28.

the space between the chosen marks is again measured by means of the auxiliary scale. Proceed in this manner until the line *R* has been reached, the reading of which is to be taken upon the overlapping scale *A* or *B*, as the case may be. In placing the scales in their successive positions, great care must be used to keep them accurately in line, and to prevent any slight displacement of each scale after it is placed in position. It is desirable to place weights upon the scales in order to reduce the liability to displacement.

The scale readings should be recorded in a systematic way so as to enable the successive distances to be easily found and added together. For example, the two readings on scale *A* at the start would be the reading of the mark *L*, and 99 (the 99 centimeter mark being chosen); the first readings on the auxiliary scale would be the reading of the 99 centimeter mark of scale *A*, and of the 1 centimeter mark of scale *B*; the two readings on scale *B* are then 1 centimeter and 99 centimeters; next come the readings on the auxiliary scale of the 99 centimeter mark of scale *B* and of the 1 centimeter mark of scale *A*; and so on until the other end of the distance is reached, and the last pair of readings, on scale *A* or *B* as the case may be, are 1 centimeter and the reading of the mark *R*. The measured distance is equal to the sum of the differences of these pairs of readings.

In making the above measurement no effort should be made to bring the divisions of the auxiliary scale into coincidence with

those of the other scales. It is easier and more accurate to place the scales at random and estimate the fractions of divisions, using a magnifying glass.

In recording the successive readings of the various scales, care must be taken to specify to which scale each pair of readings belongs, so that the measured value of the distance may be corrected for the errors of the scales as determined under (*a*) above.

Computations and results. — In adding the successive parts of the measured distance (differences of pairs of readings on successive scales) use the standardized value of the distances between the 1 centimeter mark and the 99 centimeter mark on each of the long scales; the small scale may generally be assumed to be correct, since it is used to measure so short a distance.

Find the average of all the measured values of the distance and correct this average for temperature as explained above.

Determine the probable error of the result from the departures of the individual measured values of the distance from the mean.

EXPERIMENT 13.

Measurement of Length by the Reading Telescope.

The difference in level of two points which are far from being on a vertical line may best be measured by means of the cathetometer. When, however, the two points lie approximately in a vertical line, and when it is possible to place a vertical scale near to the points as explained below, the simple reading-telescope affords a very convenient and quite accurate means for projecting the points upon the scale in order to determine their difference in level. The object of this experiment is to measure differences of level by means of the reading telescope.

Method (a). — A horizontal reading telescope is mounted on a slider which moves up and down a vertical pillar, an arrangement very similar to the cathetometer. But, unlike the cathetometer, the scale on which the readings are to be taken is placed alongside of the object to be measured. If possible, the scale should be so

near the object that both can be seen at the same time through the telescope. If this is not possible, the telescope may be turned about its vertical supporting column so as to bring first the object and then the scale into view.

To measure the required length, raise the telescope until the horizontal cross-hair coincides with the upper end of the length, then read the position of the cross-hair on the scale. Lower the telescope until the cross-hair coincides with the lower end of the length, and again read the position of the cross-hair on the scale. The difference of the two readings is the required length.

This method possesses a distinct advantage over the cathetometer, because the adjustments of the reading telescope are simpler and easier to make than those of the cathetometer, and because slight errors of adjustment (for example, an error in leveling of the telescope) generally lead to negligible errors in the result.

Method (b). — When the length to be measured is greater than the height of the vertical column upon which the reading telescope is mounted, the telescope may be tilted (instead of being raised and lowered) to take the readings. In this case the scale must be placed very near to the object, and the scale and object must be equidistant from the telescope. Furthermore the telescope should be as far from the object and scale as is compatible with accurate readings of the divisions of the scale.

Work to be done. — Measure a given vertical distance repeatedly by each method. In repeating the measurements, the position of the scale should be shifted up and down at random in order that successively estimated fractions may have different values. In making the readings it must be remembered that the telescope inverts the image.

Computations and results. — Determine the average value of the measured distance for each method (*a*) and (*b*) and find the probable error of the result in each case.

EXPERIMENT 14.

THE OPTICAL LEVER. MEASUREMENT OF ANGLE BY TELESCOPE AND SCALE.

The object of this experiment is to use the optical lever in the measurement of the thickness of a thin plate of glass, and to exemplify the use of the telescope and scale for measuring angles.

Apparatus. — The optical lever consists of a small metal table having two pointed legs at its middle and one at each end. On this table an upright mirror is mounted, its surface being parallel to the line joining the points of the two middle legs. One of the end legs is adjustable like a leveling screw, so that the points of all four legs can be made to touch the face of a true plane.

Work to be done. — (*a*) Adjust the movable leg of the optical lever until all four legs are in contact with a true plane. In making this adjustment, rock the optical lever slightly and adjust the screw until the chattering noise is no longer perceptible.

(*b*) Place a reading telescope with horizontal cross-hair and vertical scale on a level with the mirror of the optical lever, and at a distance of one or two meters therefrom; and adjust the telescope until the image of the scale is seen in the mirror with the cross-hair sighting approximately at the middle of the scale.

(*c*) Measure the distance between the mirror and the vertical scale.*

(*d*) Record the reading (*a*, Fig. 29) of the telescope and scale when the optical lever stands with its four legs in contact with the true plane. Lift one end of the optical lever (being careful not to shift the position of the other end), and slip a thin plate of glass beneath the middle legs. Take readings (*b* and *c*, Fig. 29), of the telescope and scale with the lever tilted alternately backwards and forwards. Carefully remove the plate of glass, thus

* Paper scales are frequently used with reading telescopes, and they are often one or two per cent. longer than the standard meter scale. Where great accuracy is desired the paper scale should be standardized.

bringing the lever into its original position on the true plane, and again note the reading in the telescope and measure the distance between scale and mirror to be sure that the lever has not been displaced.

(*e*) Press the lever against a smooth sheet of paper so as to make impressions of the points of the four legs. From the impression thus obtained, measure the distance between the end legs of the lever.

Computations and results.—From the averaged measured distance between scale and mirror, and the average reading of the telescope with lever tilted backwards and average reading of telescope with lever tilted forwards, compute the angle through which the lever is tilted from its level position. From this angle and half the length of the lever compute the thickness of the glass plate.

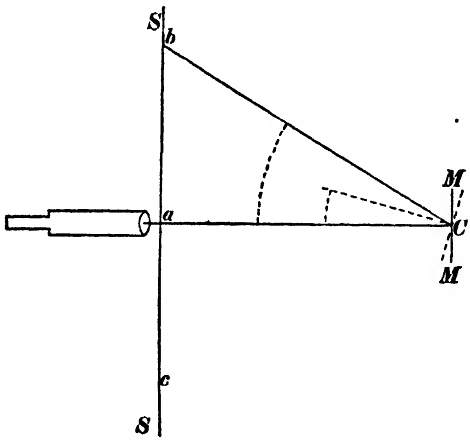


Fig. 29.

The method of computing the angle through which the mirror is turned by the tilting of the lever is as follows: The measured distance from the mirror to scale is aC, Fig. 29, a is the reading of the telescope when the lever stands with its four feet in contact with the true plane, b is the reading of the telescope when the lever is tilted backwards, c is the reading of the telescope when the lever is tilted forwards, and ϕ is the angle

through which the lever has been tilted from its horizontal position. Then

$$\tan 2\phi = \frac{a-b}{d} \quad \text{or} \quad \frac{c-a}{d}$$

It is best to calculate ϕ from the average of the two distances

$$a-b \quad \text{and} \quad c-a\left[=\tfrac{1}{2}(b-c)\right]$$

EXPERIMENT 15.

WEIGHING BY SWINGS. ELIMINATION OF ERRORS.

The object of this experiment is to weigh a body and make corrections for inequality of balance arms and for buoyancy of air.*

Work to be done. — Using the method of swings, weigh the body on each pan † of the balance, as explained on page 25. The result of these two weighings will be the two values W_r and W_l as explained on page 27. These weighings should be repeated several times in order that an estimate may be made of the probable error of the result.

Computations and results. — (*a*) Take the average of each pair of values of W_r and W_l, thus finding a series of values B_1, B_2, B_3, etc., free from the error due to inequality of arms (see page 27).

Find the average of B_1, B_2, B_3, etc., and from the departures of B_1, B_2, B_3, etc., from this average calculate the probable error of the average.

Correct the average of B_1, B_2, B_3, etc., for buoyancy of the air as explained on page 29.

Calculate the ratio of the balance arms from a pair of values of W_r and W_l as explained on page 27.

*When it is possible for the student to use a set of weights, the errors of which have been determined, he should also make corrections for errors of weights.

† The true weight (mass) of the body may be determined with the least trouble by the method of double weighing; but it is desired in the present instance to determine the ratio of the balance arms, for which purpose two complete weighings are necessary, one on each pan.

EXPERIMENT 16.

The Sensibility of the Balance.

The object of this experiment is to study the sensibility of a balance with varying loads on the pans.

Theory. — By the sensibility of the balance is meant the number of scale divisions through which the pointer is deflected for *a given small change* of weight on one pan. The sensibility is usually specified in scale divisions per milligram. The sensibility of a given balance usually has different values for different loads placed on the pans. The cause of such variation may be stated briefly as follows:

1. If the three knife-edges of the balance beam lie in the same straight line, the sensibility of the balance is not affected by the weights placed in the pans. For, if the beam is displaced from its horizontal position, one pan is lowered as much as the other is raised; the pans are still in equilibrium with each other, and no additional force is required to hold them in their new position.

2. If the straight line joining the end knife-edges passes below the middle knife-edge, the sensibility of the balance is decreased by an increase of load on the pans. For, if the beam is displaced from its horizontal position, one pan is raised more than the other is lowered, and an additional force proportional to the load on the pans will be required to hold the beam in this new position.

3. If the straight line joining the end knife-edges passes above the middle knife-edge, the sensibility of the balance is increased by an increase of load on the pans. For, if the beam is displaced from its horizontal position, one pan is lowered more than the other is raised, and the force required to hold the beam in this new position is lessened by an amount proportioned to the load on the pans.

Furthermore, an increase of load on the pans has a tendency to bend the beam, so that a balance may fulfil one of the above conditions for one load and another for another load.

Work to be done. — Using loads of 0, 20 grams, 40 grams, and so on up to nearly the maximum load for which the balance is designed (say 200 grams) take two sets of swings for each load. One set of swings is to be taken with nominally equal weights on the two pans; the other set is to be taken after placing a small additional weight on one pan. The small added weight should be of such size as to shift the position of rest of the pointer through two or three scale divisions. Each set of swings should consist of not less than four elongations to one side and three to the other, with the pointer swinging at least half the length of the scale.

In doing this work it may frequently be found that the errors of the weights are great enough to throw the pointer of the balance off the scale. When this is the case, compensate the errors of the weights by placing small weights on one pan in order to equalize the loads on the pans. In plotting the curve, use the nominal values of the weights, as their errors are so small as to have negligible effect upon the sensibility.

Calculations and results. — To find the various values of sensibility, average each set of swings as described on page 26 and take the difference between the two positions of rest thus found for each load. This difference divided by the corresponding small change of weight is the sensibility. Plot a curve, using loads as abscissas and the corresponding sensibilities as ordinates. From this curve the sensibility for any load may be found and used in weighing.

EXPERIMENT 17.

Errors of a Set of Weights.

The determination of the errors in a set of weights is usually made in terms of a single standard weight. It is most convenient to have the standard weight of such value as to be very nearly equal to the sum of the set to be standardized. Thus, given a standard 100 gram weight and a set of weights of the following

nominal values: 50, 20, 10, 10′, 5, 2, 1, 1′, 1″, (or 2, 2′, 1),* making a total of 100 grams. As many weighings must be made as there are weights in the set; inasmuch as each weighing represents an equation among the weights, and there must be as many equations as there are unknown quantities to be determined.

Let the unknown real values of the weights be designated by letters as follows:

Nominal value,	50	20	10	10′	5	2	1	1′	1″
Real value,	A	B	C	D	E	F	G	H	I

By the method of double weighing, page 28, make the nine weighings as represented by the following equations:

(1) $100 \text{ gr.} = A + B + C + D + E + F + G + H + I \pm a$

(2) $A = B + C + D + E + F + G + H + I \pm b$

(3) $B = C + D \pm c$

(4) $C = D \pm d$

(5) $D = E + F + G + H + I \pm e$

(6) $E = F + G + H + I \pm f$

(7) $F = G + I \pm g$

(8) $G = I \pm h$

(9) $H = I \pm i$

The values of the differences, a, b, c, etc., are known from the weighings. Therefore these equations may be solved for the true values of the weights. The simplest method of solution is that of substitution — substituting the value of G from equation (8) in equation (7), the values of F, G, and H from equation (7), (8) and (9) in equation (6), and so on. In this manner the right-hand members of the equations are reduced to terms of I and the known differences a, b, c, etc., and equation (1) becomes

* If the set contains two 2 grams and one 1 gram instead of one 2 gram and three 1 grams, it will be necessary to borrow a 1 gram from another set, or use the small weights to give the required number of equations.

$$100 \text{ gr.} = 100\, I \pm m$$

where m is the quantity found by adding up the known differences as the substitutions are made. The value of I is thus known, and it may be substituted in the remaining equations to give the values of the other weights. It will be noted that equation (1) above gives

$$\text{whole set} = 100 \text{ gr.} \mp a$$

Having solved for the values of the several weights, add up these values and compare with the value given by equation (1), to make sure that no error has been made in solving the equations.

Remark.— Since the weights all have the same density, the weighings need not be corrected for the buoyancy of the air.

EXPERIMENT 18.

Determination of the Volume of a Flask by Weighing.

The object of this experiment is to determine the cubic contents of a flask by weighing the flask full of a liquid of which the density is known.

Work to be done. — Using the method of double weighings, as explained on page 28, weigh the bottle empty and dry, and weigh it again when it is filled with pure distilled water at a known temperature.

Precautions.— The flask must be perfectly clean. For this purpose rinse it with caustic soda, then with hydrochloric acid, and then with common tap water, and lastly several times with distilled water. To dry the flask, blow air into it through a tube heated in the flame of a bunsen burner. The flask must be allowed to cool to room temperature before it is weighed.

The water to be used must not only be distilled water, but it must be freshly boiled to expel any air it may have absorbed, and then cooled as quickly as possible to room temperature by placing the containing vessel in a bath of cold water. The cooling distilled water must not be stirred, because of the rapid reabsorption of air. Great care must be taken to fill the flask exactly to

the reference line. In some flasks the reference line is etched around the neck, and in others it is fixed by an accurately ground stopper having a very fine hole through it as shown in Fig. 30.

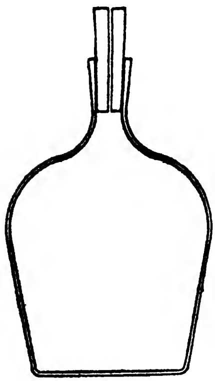


Fig. 30.

In the latter case, fill the flask completely, neck and all, then press the stopper slowly but securely into place, and wipe off the superfluous water which flows out through the hole in the stopper. Handle the flask by the neck to avoid changes of temperature due to contact with the hands. The temperature of the water in the flask may best be determined by standing the flask in a vessel of water for some minutes before pushing the stopper into position, the temperature of the water bath being taken by an accurate thermometer. *It is necessary that this temperature be very slightly greater than the room temperature in order that the water may not expand and overflow while the weighings are being made.*

Computations and results. — Subtract the weight (mass) of the dry empty bottle from the weight (mass) of the bottle filled with

TABLE.

DENSITIES AND SPECIFIC VOLUMES OF WATER.

Temperature.	Density.	Volume.	Temperature.	Density.	Volume.
—10° C.	0.99815	1.00186	20°	0.998252	1.001751
— 8	0.99869	1.00131	25	0.997098	1.002911
— 6	0.99912	1.00088	30	0.995705	1.004314
— 4	0.99945	1.00055	35	0.994098	1.005936
— 2	0.99970	1.00031	40	0.99233	1.00773
0	0.999874	1.000127	45	0.99035	1.00974
+ 1	0.999930	1.000071	50	0.98813	1.01201
2	0.999970	1.000030	55	0.98579	1.01442
3	0.999993	1.000007	60	0.98331	1.01695
4	1.000000	1.000000	65	0.98067	1.01971
5	0.999992	1.000008	70	0.97790	1.02260
6	0.999970	1.000030	75	0.97495	1.02569
7	0.999932	1.000068	80	0.97191	1.02890
8	0.999881	1.000119	85	0.96876	1.03224
9	0.999815	1.000185	90	0.96550	1.03574
10	0.999736	1.000265	95	0.96212	1.03938
15	0.999143	1.000858	100	0.95863	1.04315

water, and correct this result for buoyancy of air. This gives the true weight (mass) of the water contained by the bottle. Calculate the cubic contents of the flask from this net weight (mass) of water, taking the density of the water from the accompanying table.

EXPERIMENT 19.

DETERMINATION OF THE DENSITY OF A LIQUID.*

The object of this experiment is to determine the specific gravity of a liquid by the use of the specific gravity bottle.

Work to be done. — The object in view in the present experiment is to determine the ratio of two weights (masses). This being the case, the errors of weighing due to inequality of balance arms is without effect on the result, provided all of the weighings are carried out on a particular pan of the balance. Errors due to buoyancy of air, however, must be eliminated.

Using the method of swings, weigh the bottle empty and dry, weigh it again when it is filled with pure distilled water at a known temperature *t*, and weigh it again when it is filled with the given liquid at the same temperature.

In doing this work, take all the precautions mentioned in Experiment 18.

Computations and results. — Subtract the weight (mass) of the dry bottle from the weight (mass) of the bottle full of water, and from the weight (mass) of the bottle full of liquid; and correct these net weights for buoyancy of air. From these results, calculate the specific gravity of the liquid at the given temperature; then, taking the density of the water from the table given under Experiment 18, calculate the density of the liquid at the given temperature.

* This experiment involves a duplication of some of the observations involved in experiment 18, and it may be convenient to have the student perform the two experiments as one.

EXPERIMENT 20.

Determination of the Density of a Solid.

The object of this experiment is to determine the specific gravity of a solid by weighing in air and in water.*

Work to be done. — The object in view in the present experiment is to determine the ratio of two weights (masses). This being the case, the errors of weighing due to inequality of balance arms is without effect on the result, provided all of the weighings are carried out on a particular pan of the balance. Errors due to buoyancy of air, however, must be eliminated.

Using the method of swings, weigh the body in the ordinary way, and then suspend it in pure distilled water at a known temperature and weigh it again. The water used should be freed from air, as explained in Experiment 18, and the fine bubbles which tend to cling to the body when it is submerged should be brushed off with a camel's hair brush.

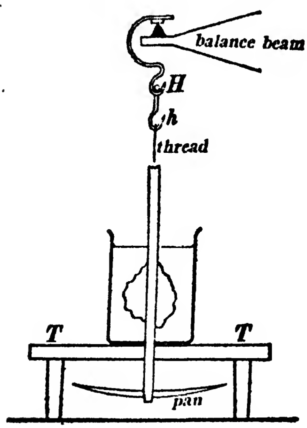


Fig. 31.

To weigh the body suspended in water, place a small table TT across the balance pan, place the vessel of water on this small table and hang the body from the hook h, which is provided for this purpose just below the point of attachment H of the pan, as shown in Fig. 31. The weight of the thread must be determined so that the net weight of the submerged body may be found.

Computations and results. — Correct the weight of the body in the air for the buoyancy of the air, as explained on page 29. When the body is submerged in water it is buoyed only by the

* But few solid substances are sufficiently definite in their physical character to warrant an accurate determination of density. Thus, iron varies greatly in density according to its chemical composition and temper, and the same is true of almost all solids, except pieces of pure crystals.

water; but the weights are buoyed by the air as before. Therefore the correction for buoyancy of air is in this case to be made by letting the term $B\lambda/\Delta$ equal zero. In making these corrections it is necessary to know the density Δ of the body. The approximate value of Δ computed from the uncorrected weighings will serve for this purpose.

Subtract the corrected net weight of the body in the water from the corrected weight in the air, and calculate the specific gravity of the body from these results.

From the specific gravity of the body thus obtained, calculate its density at the observed temperature, taking the value of the density of water at the observed temperature from the table given under Experiment 18.

EXPERIMENT 21.

The Study of the Rate of a Clock.

A clock which is used for the accurate measurement of time intervals is always adjusted so that its readings indicate very nearly the true values of elapsed time. It is impracticable, however, to adjust a clock with the accuracy that is required in some time measurements, especially for the time measurements involved in astronomical work. In accurate work, therefore, the rate of the clock (that is, the number of seconds gained or lost per day), is determined, and the apparent value of any time interval as measured by the clock is corrected to allow for the rate. The object of this experiment is to study the rate of a watch.

Work to be done. — Using a good clock as a standard, observe and record the error of a watch at a certain time each day for a series of days, and plot a curve, the abscissas of which represent days and the ordinates represent the errors of the watch. In observing the error of the watch one person should look at the standard clock and make a sharp signal at a given clock reading, and another person should observe the reading of the watch.

The watch should be wound regularly at a definite time each day to give the best results as a time-keeper. Changes of tem-

perature and more or less violent mechanical shocks affect the rate of the watch, but these things cannot be easily controlled if the watch is carried about during the test.

EXPERIMENT 22.

The Eye and Ear Method for Observing Time Intervals.

The object of this experiment is to give practice in the performance of the eye and ear method of observing time intervals, the immediate object being to determine the period of oscillation of a torsion pendulum.

Apparatus. — A heavy metal disk is suspended by a steel wire with its axis in a vertical position, constituting a torsion pendulum which makes one complete vibration in not less than 20 seconds.

A sharply defined white vertical line is marked on the edge of the disk, and a reading telescope is sighted at this vertical line.

Work to be done. — Bring the torsion pendulum to rest, and adjust the telescope so that its cross-hair coincides with the white mark on the edge of the disk. Then set the pendulum vibrating through an angle of not more than 30°, being careful to avoid pendulous motion of the disk.

Preliminary practice observations. — Glance at the clock, note and record the hour and minute, and begin to count the seconds correctly. Look into the telescope, continuing the count of seconds by listening * to the beats of the pendulum, and observe the clock readings of a number of successive transits of the white mark past the cross-hair of the telescope. Each clock reading is recorded by simply setting down the number of seconds counted up to the instant of transit. This number of seconds, added to the hour and minute recorded above, gives the complete clock reading. Where several consecutive clock readings are to be taken, as in this experiment, the counting is kept up continuously until the series of readings is complete, each recorded number

* It is generally desirable to use a telegraph sounder which is connected electrically to the clock pendulum and arranged to give a sharp click for each beat of the pendulum.

giving the number of seconds from the one observed hour and minute. Usually the transit does not coincide with a beat of the clock, and the fraction of a second is to be estimated and recorded. After a little practice, this can be done by keeping in mind how far the white line was from the cross-hair at the instant of the beat which occurs just before the transit, observing how far beyond the cross-hair the white line is at the next beat, and estimating the fraction of a second in terms of these distances. For example, let *a*, Fig. 32, be the position of the white line at the beat just before transit, *b* the position of the line at the next beat, and *c* the position of the cross-hair. The distance *ac* may be estimated as, say, 0.4 of the distance *ab*, so that, if the two beats were number 156 and number 157, respectively, the time of transit was 156.4 seconds.

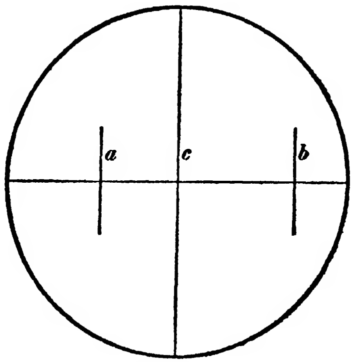


Fig. 32.

Determination of period of oscillation of torsion pendulum. — The error of a single observed time of transit in the above arrangement is quite large; so that, if the period of oscillation is to be determined with accuracy, a method must be devised in which a large number of observations may be used. The simplest scheme is the so-called "method of middle elongations," as follows: Observe and record the clock readings of *ten successive transits in both directions*, the first observed transit being from *left to right*. Allow the pendulum to vibrate undisturbed for five or ten minutes, then take another set of transits as before, the first transit being from *left to right* as in the first set. It is desirable to duplicate these two sets of readings.

Computations and results. — Consider one of the double sets of readings above obtained. The arithmetical mean of the first set of ten readings gives a fairly accurate value of the time (E)

of the middle elongation for the first ten transits. Middle elongation means the time at which the pendulum is at its greatest displacement to the right between the fifth and sixth transits. In like manner, the mean of the second set of ten readings gives the time (E') of the middle elongation for the second set of ten transits. The torsion pendulum is in the same position at both instants E and E', namely, at its greatest distance to the right. Therefore, an exact whole number N of vibrations of the pendulum occurs during the time interval $E' - E$, so that the interval $E' - E$ divided by the period τ must give a whole number. If the time interval $E' - E$ be divided by an approximate value of τ the result will be an approximate value of N, and if the approximate value of τ is not too greatly in error, the true value of N may thus be determined, since it is known that N is a whole number. A value of τ sufficiently accurate for this purpose may be obtained from the above observations as follows: Subtract the first reading of each set of transits from the ninth and the second reading from the tenth, average these remainders and divide this average by 4, which is the number of vibrations between the first and ninth and between the second and tenth readings of each set. Divide the interval $E' - E$ by this approximate value of τ, take the whole number nearest to this quotient as the true value of N, and then divide the interval $E' - E$ by this true value of N to get the accurate value of τ.

EXPERIMENT 23.

THE CHRONOGRAPH.

The object of this experiment is to afford practice in the use of the chronograph in measuring time intervals, and also to determine the observer's personal error in the use of the chronograph.

Theory. — The theory of the chronograph and the manner in which it is used in measuring time intervals is fully discussed on page 31.

Personal error. — The instant of a signal as recorded on a chronograph is always too late on account of the time required for the nerve impulse to travel from the eye to the brain and the time required for the motor impulse to travel from the brain to the muscles of the arm and hand. The interval of time which elapses between the actual instant of the signal and the recorded instant on the chronograph cylinder is called the *personal error* of the observer and it is the time required for the observer to see the signal and move his hand. The personal error of an observer is fairly constant when the signal is preceded by a warning, and it is more or less irregular and somewhat larger in value when the signal is not preceded by a warning. The method employed in this experiment for the determination of the personal error consists in the employment of a visual signal combined with an automatic device for closing an electric circuit at the instant of the signal, the signal being recorded by pressing the key and

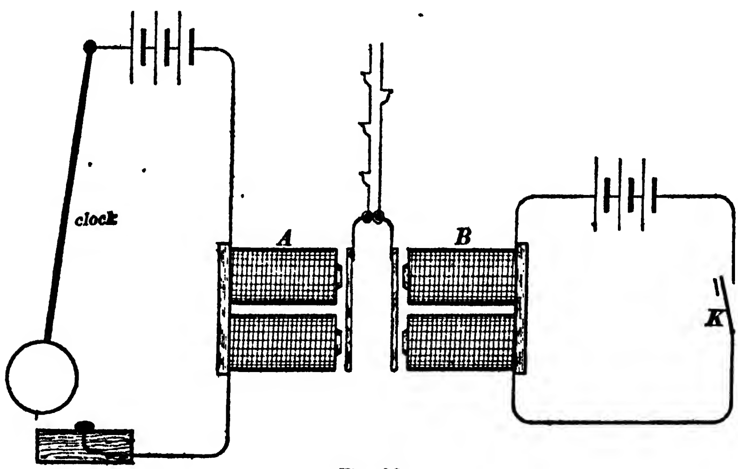


Fig. 33.

the automatic contact device being arranged to produce a record of the true instant of the signal.

Apparatus. — For the purpose of this experiment a chronograph with two recording pens is desirable. One recording pen is controlled by a contact key which is operated by hand and the other

recording pen is arranged to be controlled either by a clock pendulum or by a contact device mounted on a torsion pendulum, as described below. When the chronograph is used for recording the clock reading of a signal, the two recording pens A and B are connected as shown in Fig. 33.

The contact device on the torsion pendulum is arranged as follows: A heavy disk of metal is suspended by a steel wire forming a torsion pendulum. A short piece of platinum wire is attached to the edge of the disk and arranged to make contact with a small globule of mercury resting on top of a stationary iron block. A switch is used to facilitate the connection of the electro-magnet A to the clock or to the torsion pendulum at will. On the edge of the torsion-pendulum disk is a vertical line which is viewed through a reading telescope as in Experiment 22.

Work to be done. — The pens are to be lifted from the paper, except when a record is being made. In starting the chronograph, wait a moment to allow the chronograph to gain its full speed, then lower the pens, then close the electric circuits which actuate the pens.

(1) Wrap a sheet of paper tightly around the chronograph drum, and paste the overlapping ends of the paper with photo-library paste. The paper should overlap so that the ruling pens may glide over without catching. Fill the pens, place the pen carriage at the extreme left-hand end of the drum, start the chronograph, and test the pens to see that they are inking properly.

(2) With the torsion pendulum at rest, sight a reading telescope upon the white mark on the edge of the disk, and set the mercury drop to the position of contact with the platinum point. Then set the torsion pendulum vibrating through an angle of about 30 degrees, being careful to avoid any side to side motion of the suspending wire.

(*a*) *To determine the vibration period of the torsion pendulum.* — Start the chronograph and connect the electromagnet A to the

clock. Then, looking through the telescope, depress the key K at each transit of the white mark across the cross-hair of the telescope. Continue this for ten minutes or more.

(*b*) *Personal error when warning is given.*—Shift the pen carriage a short distance to the right in order to separate this record from the preceding. Start the chronograph and connect the pen A to the clock for ten or twenty seconds, in order to obtain a measure of the length of a second. Then shift the switch so as to connect the pen A to the torsion pendulum and make a record of transits as in (*a*) above. In this case the observer has warning of the time of transit, inasmuch as the white line can be seen moving across the field of the telescope toward the position of transit. At the end of this record again connect the pen A to the clock for ten or twenty seconds.

(*c*) *Personal error when no warning is given.*—Place between the telescope and the torsion pendulum a screen having in it a narrow vertical slit. This screen must be placed quite near to the torsion pendulum and so located that the telescope is sighted upon the slit; that is, so that the white line on the disk shall be seen through the slit only at the instant of transit. Having arranged the screen in this manner, make a record as in (*b*) above.

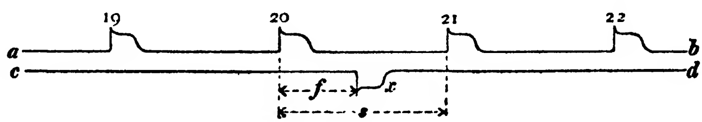


Fig. 34.

Results.—(*a*) Number consecutively the kinks in the second's pen record and also the kinks in the record of transits. The time of a given transit in seconds may then be determined as follows: Let the kink x, Fig. 34, represent the transit to be determined. The time of this transit is then 20 seconds plus the fraction f/s of a second.* In this way determine the times of

* This time is, of course, counted from the time of beginning the record. If the actual clock reading of the transit is desired (as in astronomical work) the clock reading of one second's kink in the record must be noted. In cases where it is not possi-

the first ten transits of the set and of another group of transits near the end of the set. This second group should, for convenience, be chosen so as to include transits number 21 to 30, or 31 to 40, or 41 to 50, etc., as may be possible. Suppose the group 31 to 40 to be the last one available. Subtract the time of number 1 from the time of number 31, of 2 from 32, of 3 from 33, etc., and divide these remainders by 30. This gives ten determinations of the vibration period of the torsion pendulum. Average these ten values and determine the probable error of the result.

(*b*) Determine the length of one second's space on the record sheet by measuring across ten or more second's spaces at the beginning and at the end of the record. In terms of this length measure the distance between each kink of the automatic record and the corresponding kink of the hand-made record. This gives the errors in the recorded times of the signals. Find the average of these errors and find the probable error* of the average.

(*c*) Determine the value of the personal error in this case and the probable error of the result in the same manner as under (*b*) above.

EXPERIMENT 24.

The Determination of Gravity.

The object of this experiment is to determine the acceleration of gravity and to afford an exercise in the performance of the method of coincidences for determining the time of oscillation of a pendulum.

Theory. — The most accurate determination of the acceleration of gravity is made by means of the pendulum. If a pendulum

ble to make a continuous record, two records may be made, the actual clock reading of each being determined. If the result sought is a vibration period, as above, the method of middle elongations (see Experiment 22) may then be applied.

*The fact that the thing which is measured is an error (a systematic error) must not be allowed to obscure the fact that the observations are subject to erratic error and that the probable error of the result is a measure of the probable magnitude of this erratic error.

could be constructed with a small but very heavy bob so that the mass of the supporting cord or rod could be neglected, then the acceleration of gravity would be given by the equation

$$g = \frac{4\pi^2 l}{t^2} \qquad \text{(i)}$$

where l is the length of the pendulum and t is the time of one complete oscillation in seconds.

Method (*a*). — The ideal conditions above mentioned can be approximately realized by using a small metal sphere suspended by a cord, the length of the pendulum being taken as the distance from the point of support to the center of the sphere. This method is due to Borda.

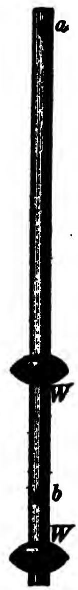


Fig. 35.

Method (*b*). — The necessity of constructing a pendulum in which the mass is concentrated approximately at a point is obviated by the use of the reversion pendulum. This was devised by Henry Kater in 1818. A simple form of Kater's pendulum is shown in Fig. 35. A metal rod is provided with two knife edges a and b from either of which it may be swung as a pendulum, and two weights WW which may be adjusted until the period t of one vibration of the pendulum is approximately the same whether the pendulum be swung from a or b. If the time of vibration is exactly the same whether the pendulum be swung from a or b, and if the center of mass of the pendulum is not half way between a and b, then the distance l between the knife edges a and b may be used in equation (i) to calculate the value of gravity; but it is practically impossible to adjust the weights WW with sufficient accuracy to enable one to use this simple scheme.

Let t be the time of vibration of the reversion pendulum when swung from knife edge a, t' the time of vibration when swung from knife edge b, let l be the accurately measured distance between the knife edges, and let x be the approximately

measured distance from knife edge a to the center of mass of the pendulum. Then

$$g = \frac{4\pi^2(2lx - l^2)}{xt'^2 - (l - x)t''^2} \qquad \text{(ii)}$$

Work to be done. *Method* (a). — A small metal sphere is suspended as a pendulum in front of a clock, and adjusted until it makes very nearly one oscillation per second, the same as the clock pendulum.

Set the given pendulum oscillating through an amplitude not to exceed 10 or 15° and note the clock readings of successive instants when the given pendulum and the clock pendulum come into exact phase with each other. Let k be the difference between the first and second clock readings expressed in seconds, then $k \pm 1$ is the number of vibrations of the given pendulum during k seconds, the algebraic sign being determined by noting whether the given pendulum gains or loses on the clock pendulum. Let m be the difference between the first and third clock readings, then $m \pm 2$ is the number of oscillations of the given pendulum in m seconds. Let n be the difference between the first and fourth clock readings, then $n \pm 3$ is the number of oscillations of the given pendulum during n seconds, and so on. Using the first and last clock readings in this way, a very exact value of the time of vibration of the given pendulum may be determined. This method of determining the time of vibration of a pendulum is called the method of coincidences.

Place a scale vertically alongside of the given pendulum and measure the distance from the point of support to the top and to the bottom of the small metal sphere, using the reading telescope as explained in Experiment 13.

Method (b). — Swing the reversion pendulum from knife edge a and determine its time of vibration t by the method of coincidences. Then swing the pendulum from knife edge b and determine its time of vibration t' by the method of coincidences. Balance the pendulum horizontally on a sharp edge to determine

the approximate position of its center of mass, and measure the distance x from the center of mass to the knife edge a. Then measure with the utmost accuracy, the distance l between the knife edges a and b.

Computations and results. — Determine the distance from the point of support to the center of the metal sphere from the distances measured under method (*a*) above, and calculate the acceleration of gravity from equation (i).

Calculate the value of g from the data obtained under method (*b*) above, using equation (ii).

Remark. — The student should carry out but one of the above methods, making repeated observations to eliminate erratic errors and in order to be able to estimate the probable error of the result. If a reversion pendulum is available, method (*b*) is very much to be preferred. The coincidences of phase may be indicated with great precision if the clock pendulum and the reversion pendulum are both arranged to make electrical contact as they pass through their vertical positions, and each arranged to operate a telegraph sounder, so that each pendulum may produce a sharp click each time it passes through the vertical position.

PART II.

EXPERIMENTS IN MECHANICS.

LIST OF EXPERIMENTS.

25. Specific gravity by Jolly's balance.
26. Specific gravity by Hare's method.
27. Use of the hydrometer.
28. Location of the center of mass of a wheel.
29. Study of a tackle block, with determination of efficiency.
30. Efficiency of a water motor.
31. Study of the drop hammer.
32. Study of harmonic motion.
33. Velocity of bullet by ballistic pendulum.
34. Moment of inertia of a body by swinging it as a gravity pendulum.
35. Moment of inertia by torsion pendulum.
36. Stretch modulus, elastic limit, and tensile strength.
37. Stretch modulus by flexure.
38. Slide modulus by torsion.
39. Slide modulus by torsion pendulum.
40. Calibration of a pressure gauge.
41. Measurement of altitude by the barometer.
42. Adjustment and testing of a spirit level.
43. A study of the compressibility of air.
44. A study of sliding friction.
45. Surface tension of water. Study of capillarity.
46. Coefficient of viscosity of a liquid.
47. The Venturi water meter.
48. Relative densities of gases by efflux.

EXPERIMENT 25.

Specific Gravity by Jolly's Balance.

The object of this experiment is to afford an exercise in the use of Jolly's balance.

Apparatus. — Jolly's balance is a delicate spring-scale conveniently arranged for weighing a body in air and in water. Two forms of Jolly's balance are in common use. In the simpler form, the upper end of the spring is attached to a fixed support, two scale pans are suspended at the lower end of the spring, one above the other, and the lower pan is always kept immersed to a particular point in a tumbler of water which stands upon an adjustable shelf. The elongation of the spring is observed by means of a needle point which is attached just above the pans and which plays over a vertical scale. This needle point is usually at a distance from the vertical scale, and in order to avoid errors of parallax the scale is etched upon a mirror and the reading is taken with the eye held in such a position as to make the needle hide its image in the mirror. In this form of instrument, the reading of the needle is taken with both pans empty, the lower pan being submerged to a particular point in the tumbler of water; the body of which the specific gravity is to be determined is then placed upon the upper pan (in air), the tumbler is moved downwards until the lower pan is submerged to the same point as before and the reading is again taken; and then the body is placed upon the lower pan (in water), the tumbler is adjusted up and down until the lower pan is submerged to the same point as before, and the reading is again taken.

In the more elaborate form of Jolly's balance the upper end of the spring is attached to an adjustable post which is moved up or down by means of a rack and pinion until the pans come to a marked position, and the position of the adjustable post (elongation of spring) is read from a vertical scale. In this form of instru-

ment the tumbler in which the lower pan is submerged need not be adjusted, inasmuch as the suspended pans are always brought to a standard position before the reading is taken.

The readings above specified are adapted to the determination of the specific gravity of a solid. To determine the specific gravity of a liquid, a glass sinker is suspended on the spring and the following readings are taken:

(*a*) with the sinker suspended in air,

(*b*) with the sinker submerged in water, and

(*c*) with the sinker submerged in the given liquid.

Work to be done. — Determine the specific gravities of several solids and liquids. Each reading should be repeated several times. Care must be taken to avoid friction between the pans and surrounding objects, and every precaution used to free the submerged body from air bubbles.

EXPERIMENT 26.

Specific Gravity by Hare's Method.

The object of this experiment is to determine the specific gravity of a liquid by measuring the heights of balanced columns of the given liquid and of water.

Apparatus. — The apparatus used is shown in Fig. 36. The heights of the two liquid columns W and L are inversely proportional to the densities of the respective liquids. Therefore

$$S = \frac{W}{L}$$

where W is the height of the water column, L is the height of the balanced column of the given liquid, and S is the specific gravity of the liquid.

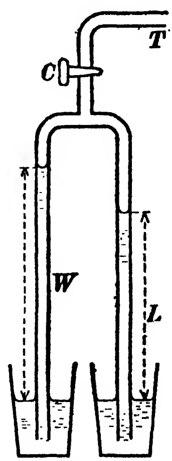


Fig. 36.

Work to be done. — Determine the specific gravities of several liquids in the manner described above, making two or more complete determinations for each

liquid. Rinse the tube with distilled water before each set of observations and also when the work is finished. It is desirable to use only aqueous solutions of salts in order to avoid the trouble involved in cleaning the tube after it has been filled with oil.

DENSITIES OF SALT SOLUTIONS AT 18° C.

(GRAMS PER CUBIC CENTIMETER.)

Percentage strength means in each case the number of parts by weight of the anhydrous salt or acid in a hundred parts by weight of the solution.

%	KOH	KNO_3	K_2CO_3	$K_2Cr_2O_7$	NH_3	NH_4Cl	NaCl	Na_2CO_3	%
0	0.999	0.9986	0.9986	0.999	0.999	0.9986	0.9986	0.999	0
5	1.045	1.0305	1.0442	1.035	0.978	1.0142	1.0345	1.051	5
10	1.092	1.0632	1.0910	1.072	0.958	1.0289	1.0711	1.104	10
15	1.114	1.097	1.140	1.109	0.940	1.0430	1.1090	1.159	15
20	1.190	1.133	1.191		0.923	1.0571	1.1485		20
25	1.240		1.244		0.908	1.0710	1.1897		25
30	1.293		1.299		0.893				30
35	1.347		1.356		0.881				35
40	1.405		1.415						40
45	1.464		1.477						45
50	1.53		1.541						50
55	1.59								55
60	1.66								60

%	$BaCl_2$	$ZnSO_4$	$CuSO_4$	HCl	HNO_3	H_2SO_4	Alcohol.	Sugar.	%
0	0.999	0.999	0.999	0.9986	0.999	0.9986	0.9986	0.9986	0
5	1.044	1.051	1.051	1.0236	1.027	1.0324	0.9898	1.0183	5
10	1.093	1.107	1.107	1.0482	1.056	1.0673	0.9824	1.0386	10
15	1.147	1.167	1.167	1.0734	1.086	1.1036	0.9760	1.0597	15
20	1.204	1.232	1.23*	1.0989	1.118	1.1414	0.9696	1.0815	20
25	1.268	1.305		1.1248	1.151	1.181	0.9628	1.1042	25
30		1.379		1.1508	1.184	1.221	0.9551	1.1277	30
35				1.1757	1.217	1.262	0.9463	1.1520	35
40				1.199	1 250	1.306	0.9367	1.1773	40
45					1.283	1.351	0.9264	1.2034	45
50					1.314	1.398	0.9155	1.2304	50
55					1.344	1.449	0.9043	1.2584	55
60					1.372	1.502	0.8928	1.2874	60
65					1.397	1.558	0.8811	1.3173	65
70					1.418	1.615	0.8693	1.3482*	70
75					1.438	1.673	0.8574	1.380*	75
80					1.457	1.732	0.8452		80
85					1.473	1.783	0.8327		85
90					1.489	1.817	0.8197		90
95					1.50	1.837	0.8060		95
100					1.52	1.835	0.7911		100

* Supersaturated.

The best method for measuring the heights of the two columns W and L is by means of the reading telescope, as described in Experiment 13. Two or more readings of each height should be taken.

EXPERIMENT 27.

Use of the Hydrometer.

The object of this experiment is to afford an exercise in the use of the hydrometer for the determination of percentage strengths of solutions of salts and acids.

Apparatus. — The hydrometer consists of a glass float with a vertical cylindrical stem on which is a scale for indicating the specific gravity of any liquid in which the instrument floats.

A great variety of hydrometer scales are in common use but for this experiment the hydrometer is supposed to indicate specific gravities directly.

The indications of a hydrometer vary with the temperature, and in accurate work the liquid which is being tested should be brought to the temperature at which the hydrometer readings are correct.

In taking the reading of a hydrometer always sight along the liquid surface *through the liquid*, not through the air, thus avoiding to some extent the error due to the capillary elevation of the liquid near the stem.

Work to be done. — Determine the specific gravity of several salt solutions, and take their percentage strengths from the tables on page 96.

EXPERIMENT 28.

Location of the Center of Mass of a Wheel.

The object of this experiment is to determine the position of the center of mass of a wheel.

Apparatus. — A wheel mounted on an axle is placed upon levelling ways or tracks, as shown in Fig. 37. This figure shows

the manner in which the large fly-wheel of an engine is balanced in a shop.

Work to be done. — (*a*) Level the track upon which the wheel rests.

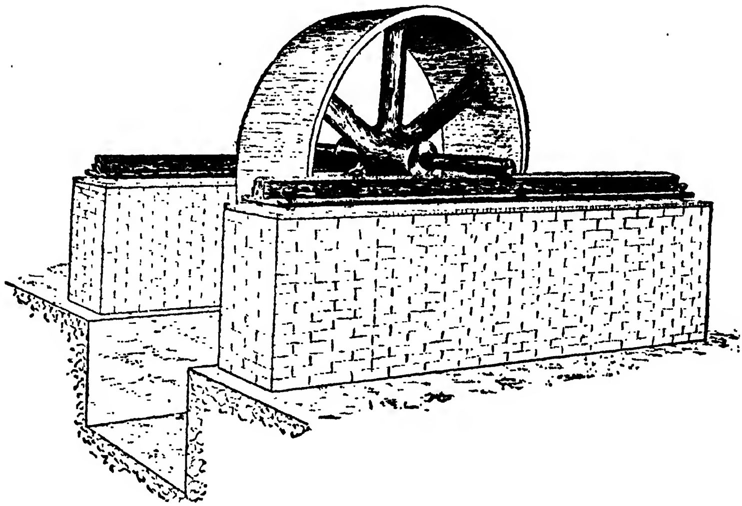


Fig. 37.

(*b*) Hang a double plumb-line over the shaft, as shown in Fig. 38, let the wheel vibrate until it comes to rest, and carefully

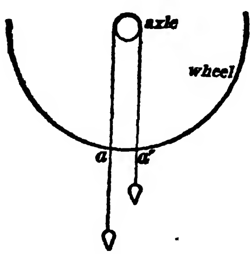


Fig. 38.

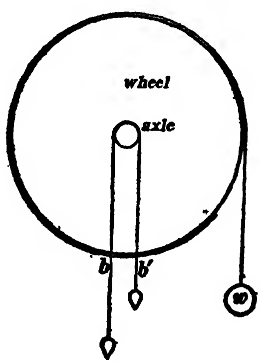


Fig. 39.

mark the points *aa'* at which the lines cross the rim of the wheel. This may be done with a sharp bit of chalk.

(*c*) Hang a known weight w from the rim of the wheel, as shown in Fig. 39. Again let the wheel vibrate and come to rest and mark the points bb' at which the plumb-lines cross the rim. Measure the arcs ab and $a'b'$.

(*d*) Repeat (*c*) with the weight w hung on the opposite side of the wheel so as to eliminate to some extent errors due to lack of level of the track.

(*e*) Weigh the wheel.

(*f*) Measure the radius of the wheel.

These observations should be repeated at least twice.

Computations and results. — Let A, Fig. 40, be the point midway between the two points aa', and B the point midway between the points bb'. Figure 40 represents the equilibrium position of the wheel when the weight w is attached to it. The line drawn from O to A passes through the center of mass of the wheel, and the distance x of the center of mass from O is determined by the equation

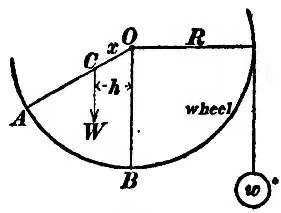


Fig. 40.

$$wR = Wh = Wx \sin \overline{AOB}$$

in which w, W, and R are known, and the angle $\overline{AOB}$ may be determined by measuring the arc AB between the marked points on the wheel.

EXPERIMENT 29.

Study of a Tackle Block with Determination of Efficiency.

The efficiency of a machine is the ratio of the work done *by* the machine in a given time to the work done *on* the machine during the same time. This is ordinarily expressed by the equation

$$\text{efficiency} = \frac{\text{output}}{\text{input}}$$

Efficiency may also be defined as the ratio of the power delivered by a machine to the power expended in driving the machine.

The object of this experiment is to determine the efficiency of a tackle block. The efficiency of the tackle block is of no practical importance, but its determination furnishes one of the simplest examples of an efficiency test.

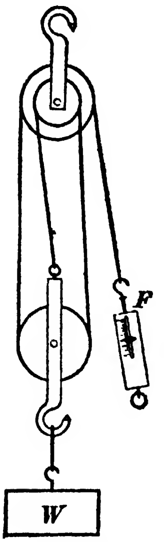


Fig. 41.

Work to be done. — Apply a spring scale as shown in Fig. 41, and measure the force F required to lift a known weight W. Determine the values of F for a series of values of W.

Computations and results. — The work done *on* the tackle block is equal to Fd where F is the force applied to the hand-rope and d is the distance through which this force acts. The work done *by* the tackle block is equal to WD, where W is a weight lifted and D is the distance through which it is lifted. The ratio d/W is equal to n, where n is the number of ropes attached to the lower block, or in general, to the moving block. Therefore, we have for the efficiency of the block

$$E = \frac{WD}{Fd} = \frac{W}{nF}$$

Calculate the efficiency for each value of W and plot a curve of which the abscissas represent values of W and ordinates represent values of E.

EXPERIMENT 30.

Efficiency of a Water Motor.

The object of this experiment is to determine the efficiency of a water motor.

Apparatus. — In testing the efficiency of a small motor, an ordinary speed counter cannot be used because of the amount of power required to drive the counter when it is pushed against the

end of the motor shaft. It is necessary, therefore, in testing a small motor to have the speed counter mounted on the brake, as shown in Fig. 42,* in which *S* is the end of the motor shaft, *P* is the motor pulley, *C* is the revolution counter, and *ll* is the brake arm. The weight *W* should be adjusted until the brake arm is accurately balanced when the pan is empty.

The water discharge from the motor is most easily measured by allowing it to collect in a cylindrical tank in which is a float

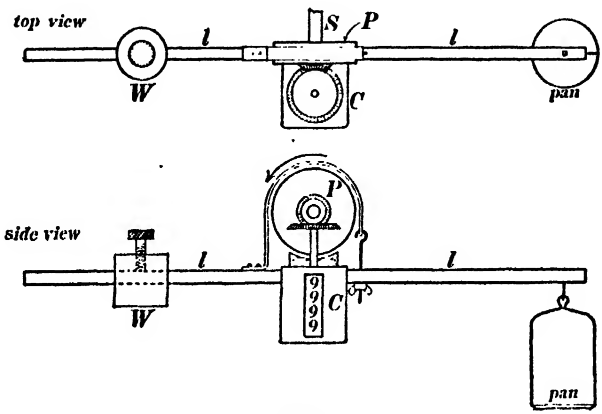


Fig. 42.

that carries a vertical stem by means of which the increase of depth of the water may be measured.

Work to be done. — Start the motor and place a desired amount of weight upon the pan which hangs from the end of the brake arm, adjust the brake clamp until the brake beam "floats," note the depth of water in the receiving tank, note the reading of the speed counter and after the lapse of a certain time interval, note the reading of the speed counter and the depth of the water in the tank a second time. The reading of the pressure gauge is to be taken at intervals during the run and the brake clamp is to be repeatedly adjusted if necessary so as to keep the brake arm floating.

* A very small helical spring of steel wire used as a flexible shaft makes the best connection from motor shaft to revolution counter.

The observations should be taken for the determination of the efficiency of the motor at a series of speeds.

Computations and results. — From the mean pressure of the water as delivered to the motor and the volume of water discharged by the motor, the total energy delivered *to* the motor may be calculated. From the number of revolutions of the motor, the length of the brake arm and the weights on the pan, the amount of work delivered *by* the motor may be calculated, whence the efficiency is known.

Plot a curve of which the abscissas represent the various speeds of the motor and the ordinates the corresponding efficiencies.

EXPERIMENT 31.

Study of the Drop Hammer.

The object of this experiment is to study the drop hammer, the immediate object being to determine the force exerted by the hammer.

Apparatus. — Figure 43 shows the drop hammer to be used. The hammer *H* slides freely along two vertical guides, and is allowed to fall upon a nail partly driven into a block of wood, as shown.

Work to be done. — (*a*) Weigh the hammer.

(*b*) Start the nail into the wood, and measure very carefully its projecting length.

(*c*) Place the block of wood and nail under the drop hammer, raise the hammer until its lower edge is coincident with a marked point on the guides, allow it to drop on the nail, and then measure the distance from the top of the nail to the marked point on the vertical guides.

(*d*) Measure the projecting length of the nail with great care.

(*e*) Repeat (*b*), (*c*), and (*d*) until the nail is driven completely into the block of wood.

(*f*) Start a second nail into the same or a similar block of wood, place the block and nail upon a platform scale as shown

in Fig. 44, force the nail into the wood by means of the screw, as shown, and take a series of readings of projecting length of nail and force exerted by screw.

Computations and results. — Let W be the weight of the hammer in pounds and H the distance from the marked point on the guides to the top of the nail *after the hammer blow.* Then WH is the work done by gravity, and, if the wooden block is on a solid foundation so that but little energy is lost in vibratory

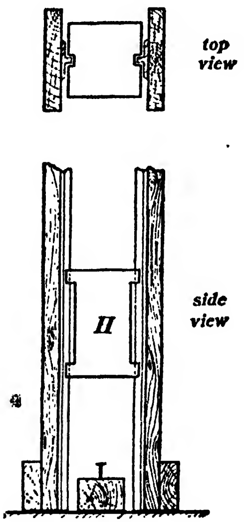


Fig. 43.

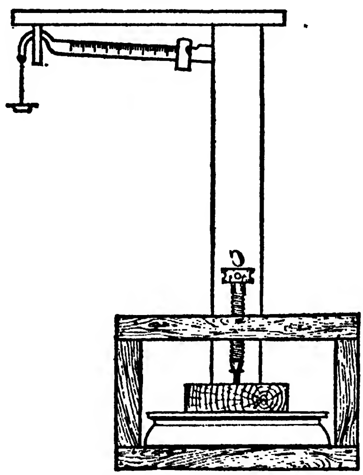


Fig. 44.

motion, this work WH is equal to the work Fd done in driving the nail, where F is the average force pushing on the nail during the hammer blow and d is the amount that the nail is driven by the blow. Therefore

$$F = \frac{WH}{d} \qquad \text{(i)}$$

Calculate the value of F in this way for each hammer blow, and plot two curves as follows: Plot the averages of the projecting lengths of the nail before and after each hammer blow as

abscissas, and the corresponding values of F as ordinates. Plot the values of F calculated from equation (i) and also the values of F as measured by the platform scale.

EXPERIMENT 32.

Study of Harmonic Motion.

The object of this experiment is to study a simple case of harmonic motion.

Theory. — The elongation e of a spring is proportional to the stretching force F; that is

$$F = ae \tag{i}$$

in which a is the proportionality constant. It is here understood that F is expressed in dynes and e in centimeters.

When a body of mass M is suspended by a spring, and the spring stretches by an amount sufficient to support the body. If the body is pulled x centimeters further *down*, the spring will pull it *up* by a force $F = ax$ in excess of the weight of the body; and if the body is pushed *up* x centimeters, the weight of the body will exceed the pull of the spring by an amount $F = ax$, so that an unbalanced force $F = ax$ will pull the body *down*. Therefore,* if the body is set vibrating up and down, it will perform simple harmonic motion, such that

$$\frac{4\pi^2 M}{t^2} = a \tag{ii}$$

where t is the period of one complete vibration.

If M and t are determined by observation, then a is known, and equation (i) enables the value of a force to be calculated in dynes when the elongation e has been measured in centimeters.

Work to be done. — (*a*) Hang a series of weights of known mass upon a spring and determine the vibration period of each. The vibration period may be determined by counting the number of oscillations in a minute or in two minutes. This may best be

* See Franklin and MacNutt, *Mechanics*, page 89.

done by holding a watch in such a position that the vibrating body can be seen while the eye is kept on the seconds hand of the watch. Make at least five determinations of each vibration period.

(*b*) Observe the elongation e of the spring for one of the heavier weights, making a record of the value of the weight in grams.

Computations and results. — Equation (ii) is not exactly correct for the reason that the mass of the spring is neglected.

To determine the constant a *of the spring.*— Calculate the value of a for each value of M and t, using equation (ii). Plot these calculated values of a as ordinates and the corresponding values of M as abscissas, and draw a smooth curve among the plotted points. This curve would be a straight line if it were not for the influence of the mass of the spring. The effect of the mass of the spring is to give a curve which approaches a straight line for the larger values of M. The ordinate of this curve for the largest value of M gives the most nearly correct value for a. Specify the unit in terms of which the constant a is expressed and explain how the spring might be used for measuring an unknown force in dynes.

To determine the acceleration of gravity. — The spring having been standardized as above, it may be used for measuring any force in dynes. Therefore, if the force with which the earth attracts M grams be measured by the spring, the value of gravity may be determined, inasmuch as the force with which the earth attracts M grams is equal to Mg dynes. Calculate the acceleration of gravity in this way from the data obtained under (*b*) above.

EXPERIMENT 33.

Velocity of Bullet by the Ballistic Pendulum.

The object of this experiment is to determine the velocity of a rifle bullet.

Apparatus. — The pendulum is arranged as shown in Fig. 45.

A bullet of mass m moving at velocity v strikes the pendulum which is at rest. Let M be the mass of the pendulum. After the impact, the bullet and the pendulum move as one body of mass $M+m$. Let V be the common velocity of bullet and pendulum after impact. Then the momentum after impact is

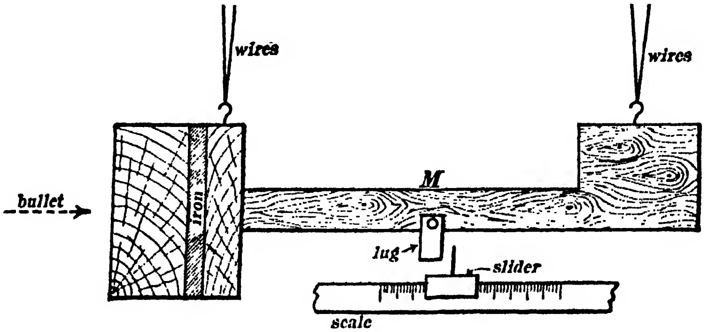


FIg. 45.

$(M+m)V$ which must be equal to the momentum mv of the bullet before impact, according to the principle of the conservation of momentum. Therefore

$$(M+m)V = mv$$

or

$$v = \frac{V(M+m)}{m} \tag{i}$$

This equation would enable the calculation of v from an observed value of V, M and m being known. The velocity V of the pendulum after impact may be determined by observing the horizontal distance d through which the pendulum swings immediately after the impact, as follows: The vertical distance h through which the pendulum rises is the same as that through which it would rise against gravity if the velocity V were vertically upwards. Therefore

$$V = \sqrt{2gh} \tag{ii}$$

where g is the acceleration of gravity. The value of h may be

determined in terms of d and the length l of the pendulum, by means of the equation

$$h = l - \sqrt{l^2 - d^2} \qquad \text{(iii)}$$

in which l is the vertical length of the suspending wires of the pendulum. Equation (iii) is evident from Fig. 46, in which figure the pendulum is represented by a small sphere M.

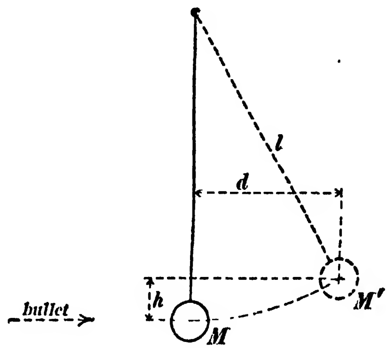


Fig. 46.

Work to be done. — Remove the body M from its suspending wires, screw on a new impact block, and weigh. Then suspend M and adjust the wires so that M is horizontal and at the same level as the gun. The axes of the gun and of M should be in the same straight line, otherwise the impact of the bullet will impart a wobbling motion to the pendulum.

Measure the vertical length of the suspending wires. Place the slider on the scale so that it is just in contact with the lug when the pendulum is at rest, and observe and record the position of the slider on the scale. The scale must be horizontal and parallel with the pendulum.

Shoot against the wooden block, aiming at the center, and observe the position to which the slider is moved.

Repeat these observations several times, cleaning the gun after every shot. If the gun is not kept clean the velocity of the bullet will become less on account of the clogging of the barrel with powder residue.

The mass of the bullet may be best determined by weighing a sample bullet.

The gun must be carefully cleaned before returning it to the supply room at the end of the experiment.

Computations and results. — Compute the velocity of the bullet from each observation and determine the probable error of the result.

EXPERIMENT 34.

Moment of Inertia of a Body by Swinging it as a Gravity Pendulum.

The object of this experiment is to determine the moment of inertia of a body about a given axis, for example, the moment of inertia of the connecting rod of an engine about the axis of the cross-head pin.

Method (a). — Suspend the body as a gravity pendulum so that the axis of suspension is coincident with the axis about which the moment of inertia is desired, and observe its periodic time of vibration. Weigh the body, balance it horizontally on a knife-edge and measure the distance x from the axis of suspension to the center of mass of the body.

The moment of inertia may then be calculated from the equation

$$\frac{4\pi^2 K'}{t^2} = mgx \qquad \text{(i)}$$

in which K' is the desired moment of inertia, t is the time of one complete vibration, m is the mass of the pendulum, x is the distance from the axis of suspension to the center of mass, and g is the acceleration of gravity.

Method (b). — It may not be convenient to swing the body as a pendulum about the desired axis. In this case, swing the body as a pendulum about any convenient axis parallel to the desired axis, and determine the moment of inertia of the body about the convenient axis by method (a).

Then calculate the moment of inertia of the body about the axis passing through its center of mass by means of the equation

$$K' = K + mx^2 \qquad \text{(ii)}$$

where K is the moment of inertia about an axis passing through

the center of mass of the body, K' is the moment of inertia about any other axis parallel to the axis through the center of mass, m is the mass of the body, and x is the distance between the axes.

The moment of inertia about the axis passing through the center of mass being thus determined, the moment of inertia about the desired axis may then be calculated from equation (ii), using for x the distance of the desired axis from the center of mass.

Fig. 47.

Work to be done. — Swing the connecting rod as a pendulum by hanging it upon a knife-edge S as shown in Fig. 47. Determine the period of vibration of the pendulum by counting the number of swings in two or three minutes of time. Balance the connecting rod horizontally on a knife edge as shown in Fig. 48, and measure the distance x. Weigh the rod.

These observations should be repeated two or three times.

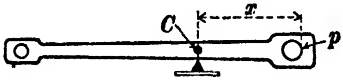


Fig. 48.

Computations and results. — Compute the moment of inertia of the connecting rod about an axis passing through the point p, Fig. 48, compute the moment of inertia of the rod about an axis passing through the point C, and compute the moment of inertia of the rod about an axis passing through the center of each pinhole.

EXPERIMENT 35.

Moment of Inertia by the Torsion Pendulum.

The torsion pendulum affords a convenient means for determining moment of inertia of a body about an axis passing through its center of mass. If the moment of inertia is desired about an axis which does not pass through the center of mass of the body, this moment of inertia may be computed by equation (ii) of Experiment 34. The object of this experiment is to determine the

moment of inertia of a wheel about an axis passing through its center of mass.

Apparatus. — The torsion pendulum to be used in this experiment consists of a light metal frame suspended by a steel wire.

The time of vibration of the empty frame is first observed. Then the wheel of which the moment of inertia is desired is placed horizontally in the frame with its axis coincident with the suspending wire, and the time of oscillation of the pendulum is again determined. The wheel is then replaced by a metal ring, Fig. 49, of which the moment of inertia is known by calculation, and the time of vibration of the pendulum is again observed. From these data the moment of inertia of the wheel may be calculated as explained below.

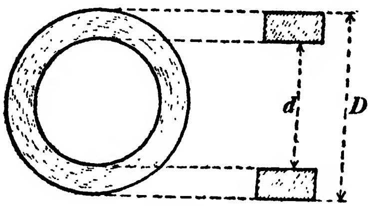


Fig. 49.

Work to be done. — (*a*) Start the empty frame oscillating about the wire as an axis through an amplitude of not more than 30°, and determine its vibration period t by obtaining the time of twenty swings, or more.

(*b*) Place the wheel in the frame, adjust it carefully to bring its axis coincident with the suspending wire, and observe the period of vibration t' in the same manner.

(*c*) Remove the wheel and put in its place the metal ring, adjust the ring so that its axis is coincident with the suspending wire, and observe the period of vibration t'' as in (*a*).

(*d*) Weigh the ring and measure its inside and outside diameters d and D.

Computations and results. — Let K be the unknown moment of inertia of the metal frame, K' the unknown moment of inertia of the wheel, and K'' the moment of inertia of the metal ring.

The value of K'' is given by the equation

$$K'' = \tfrac{1}{8}M(D^2 + d^2) \tag{i}$$

When the empty frame oscillates as a torsion pendulum, we have

$$\frac{4\pi^2 K}{t^2} = b \tag{ii}$$

in which b is the unknown constant of torsion of the suspending wire.

When the wheel is in place, we have the equation

$$\frac{4\pi^2(K + K')}{t'^2} = b \tag{iii}$$

When the metal ring is in place, we have the equation

$$\frac{4\pi^2(K + K'')}{t''^2} = b \tag{iv}$$

Using equations (ii), (iii) and (iv), the unknown quantities b and K may be eliminated and the value of K' be determined in terms of K''.

EXPERIMENT 36.

Stretch Modulus, Elastic Limit, and Tensile Strength.

The object of this experiment is to determine the stretch modulus, the elastic limit, and the tensile strength of a sample of steel or brass wire.

Theory. — The elongation e of a wire is proportional to the stretching force (Hooke's law), provided the stretching force is not excessive; and when the stretching force is relieved, the wire returns to its initial length. When, however, the stretching force is increased more and more, a limit is reached beyond which the elongation of the wire is not proportional to the stretching force, and then the wire does not return to its initial length when the stretching force is relieved.

It is convenient to express the stretching force in pounds per square inch of sectional area of the wire. When so expressed, the stretching force is called a *longitudinal stress*. The elongation of the wire is conveniently expressed as a fractional part of

the initial length of the wire. When the elongation is so expressed, it is called a *longitudinal strain.* According to Hooke's law, the longitudinal strain (elongation per unit length) which is produced by a longitudinal stress (stretching force per unit of sectional area) is proportional to the stress, and therefore the ratio of the stress to the strain is a constant for a given substance. This constant is called the *stretch modulus* of the substance and it is given by the equation

$$E = \text{stress} \div \text{strain} = \frac{F}{q} \div \frac{l}{L} = \frac{FL}{ql} \tag{i}$$

in which F is the stretching force, q is the sectional area of the wire, L is the initial length of the wire, and l is the increase of length.

The limiting value of the longitudinal stress (stretching force

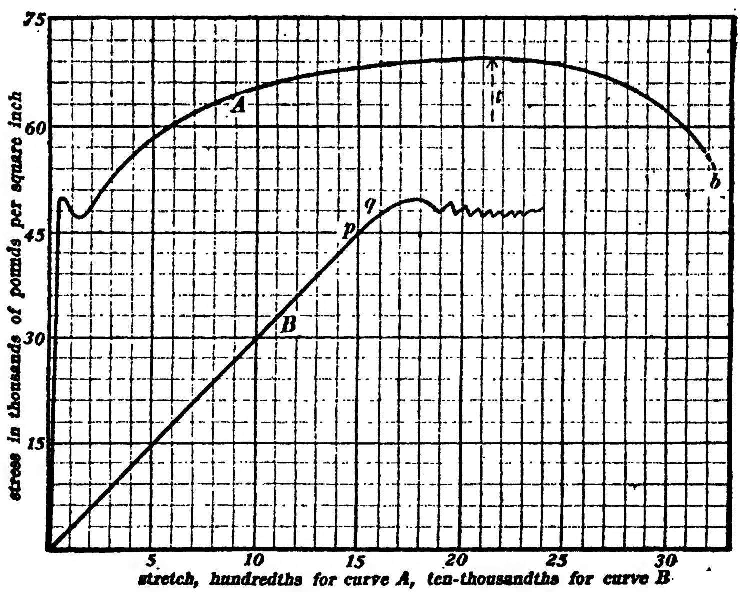


Fig. 50.

per unit of sectional area) beyond which the wire does not return to its initial length when the stress is relieved but takes what is called a *permanent set,* is called the *elastic limit* of the material.

When a wire is stretched beyond its elastic limit, the stretching force must be increased more and more, and the maximum value of the stretching force per unit of sectional area is called the *tensile strength* of the material.

A clear idea of the behavior of a steel wire when it is subjected to a continually increasing stretching force is shown in Fig. 50, in which the ordinates represent increasing values of stress (stretching force per unit of sectional area), and the abscissas represent the corresponding elongations of the wire expressed as fractional parts of the initial length of the wire. The stretch is very accurately proportional to the stress up to the point p, which marks the elastic limit of the substance. A slightly increased stress causes the wire to yield very greatly indeed, and the final maximum value of the stress, which is represented by the ordinate t, is the tensile strength of the wire.

Apparatus. — Figure 51 represents a long wooden beam with one end of the wire clamped under the screw a, and the other end of the wire attached to a spring scale which is arranged to be

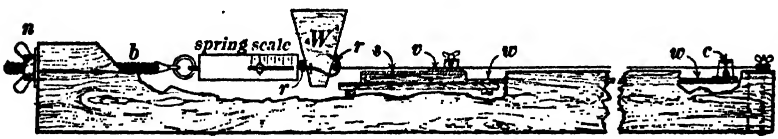


FIG. 51.

pulled out by means of the long screw b, so as to place the wire under a continually increasing stress. A wooden wedge W is arranged to drop between two cross-rods rr, so as to catch the recoil of the spring when the wire breaks. A very light wooden strip ww, which is fastened to the wire at the point c by means of a spring clip, carries a finely graduated steel scale s, and the elongation of the wire is indicated by the vernier v which is attached to the wire and plays over the scale s. The vernier v is attached to the wire at a definite point of the wire by means of a spring clip somewhat similar to the clip c.

This apparatus is not suited to the testing of soft steel, inas-

much as the elongation of a soft steel wire frequently amounts to as much as 20 per cent. before breaking occurs.

Work to be done. — (*a*) Measure the diameter of the sample of wire at a number of points by means of micrometer caliper.

(*b*) Unscrew the nut *n*, move the spring scale as near as possible to the end of the wooden strip *ww*, and connect the sample of wire to be tested, as shown in the figure. It is very important that the test wire be free from kinks.

(*c*) Turn the nut *n* until the wire is subjected to barely enough tension to hold it straight, and measure the length *L* of the wire between the point *c* and the point of attachment of the vernier *v*.

(*d*) Turn the nut *n* slowly so as to subject the wire to a continually increasing tension, and take simultaneous readings of the spring scale and of the vernier *v* until the wire breaks.

Computations and results. — (*a*) Plot a curve of which the abscissas represent the elongations *l* of the wire as determined by the readings of the vernier *v*, and of which the ordinates represent the readings of the spring scale.

(*b*) The first portion of the curve plotted under (*a*) will be a straight line. Choose the largest reading of the spring scale which lies on the straight portion of the curve, and calculate the value of the stretch modulus of the wire from this value of the reading of the spring scale and the corresponding elongation *l* of the wire.

(*c*) From the largest reading of the spring scale which lies on the straight portion of the curve (*a*), determine the elastic limit of the material of the wire in pounds per square inch.

(*d*) From the maximum reading of the spring scale calculate the tensile strength of the wire in pounds per square inch.

(*e*) Note the percentage elongation of the wire at the instant of break.

EXPERIMENT 37.

Stretch Modulus by Flexure.

The object of this experiment is to determine the stretch modulus of steel by a bending test on a bar or beam.

Apparatus. — The beam to be tested is laid across two supports SS and bent by weights WW hung at its ends as shown

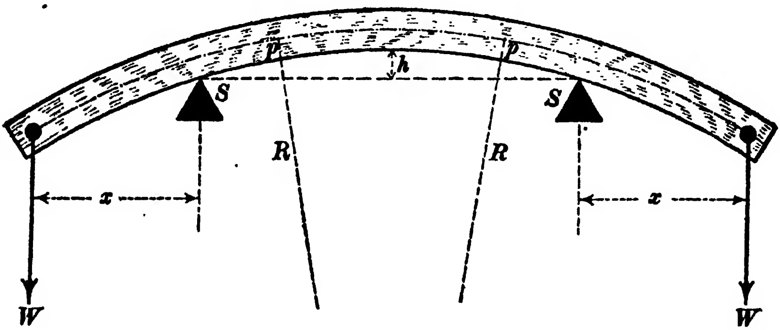


Fig. 52.

in Fig. 52. The portion of the beam between the two supports SS forms an arc of a circle. Let R be the radius of this circle measured up to the middle line of the beam. Then *

$$E = \frac{12RWx}{bd^3} \tag{i}$$

where W is the weight hung at each end of the beam, x is the distance shown in Fig. 52, b is the horizontal breadth of the beam, and d is the depth of the beam.

The value of R may be most easily determined by measuring the vertical distance h moved by the middle point of the beam when the loads WW are applied. Then

$$R = \frac{D^2}{8h} + \frac{h}{2} + \frac{d}{2} \tag{ii}$$

where D is the horizontal distance between the supports SS in Fig. 52.

* See Franklin and MacNutt, *Elements of Mechanics*, pages 176 to 180.

Work to be done. — The beam which is to be used in this test is of small cross-section, and the dimensions a and b may best be measured by means of a micrometer caliper or a vernier caliper. The distances D and x may be measured by means of a meter scale, and the value of h may be determined with sufficient exactness by measuring the distance between the middle point of the beam and the top of the table upon which the apparatus stands when the beam is loaded and when the beam is unloaded.*

Load the beam step by step with weights W, $2W$, $3W$, etc., up to the maximum safe load (see instructor), taking readings to determine the corresponding values of h. Then unload the beam step by step, and take readings to determine the corresponding values of h. These observations should be repeated several times.

Computations and results. — (a) Plot a curve of which the ordinates represent the values of the loads W, $2W$, $3W$, etc., and of which the abscissas represent the corresponding values of h, each value of h being the average of all of the observed values for a given load. This curve should be a straight line (Hooke's law) if the elastic limit has not been exceeded. Draw a smooth curve among the plotted points.

(b) Using values of W and h taken from the smooth curve (a), calculate the stretch modulus of the material of the beam using equation (i).

EXPERIMENT 38.

Slide Modulus by Torsion.

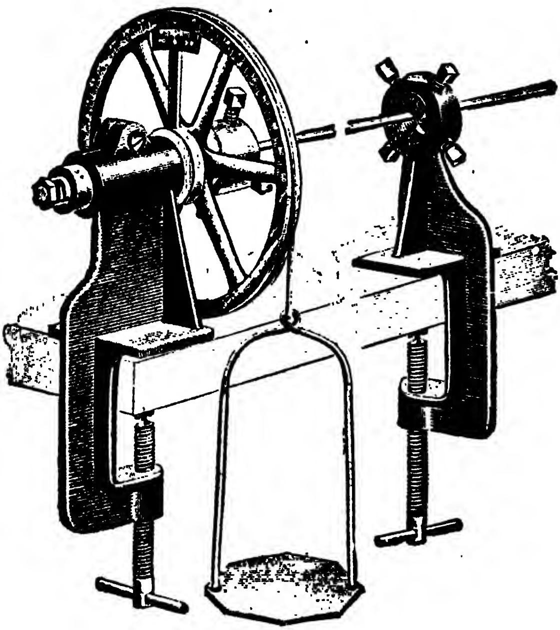
The object of this experiment is to determine the slide modulus of a material by observing the angle of twist of a rod of known dimensions when it is subjected to a known twisting force or torque.

Apparatus. — The apparatus used in this experiment is shown in Fig. 53. One end of the rod to be tested is fastened to the wheel and the other end is fastened in the fixed clamp as shown; the rod is subjected to a known twisting force by placing the

* By choosing a long slim beam the value of h can be made large enough to be measured as here explained.

weights in the suspended pan; and the angle of twist is read off the divided circle.

Work to be done. — Clamp the test rod in the testing machine, measure its length between the clamps, and measure its diameter by means of a micrometer caliper. Take the reading of the divided circle and then load the scale pan step by step, W, $2W$,



No. 663.

Fig. 53.

$3W$, etc., up to the maximum safe load (see instructor), taking readings for each load. Then unload the scale pan step by step and take the circle reading for each load. Measure the diameter of the wheel and divide by two to get the lever arm of the suspended pan.

These observations should be repeated several times.

Computations and results. — (a) Plot a curve of which the ordinates represent the values of the loads W, $2W$, $3W$, etc., and of which the abscissas represent the corresponding angles of twist

of the rod, each angle of twist being the average of all of the observed values for the given load. This curve should be a straight line if the elastic limit has not been exceeded. Draw a smooth curve among the plotted points.

(*b*) Using values of W and of angle of twist taken from the smooth curve (*a*), calculate the slide modulus n of the material of the rod from the equation

$$n = \frac{2LT}{\pi r^4 \theta}*$$

where L is the length of the rod between clamps, r is the radius of rod, T is the twisting force or torque, and θ is the angle of twist expressed in radians.

EXPERIMENT 39.

Slide Modulus by Torsion Pendulum.

The object of this experiment is to determine the slide modulus of a sample of steel in the form of wire.

Apparatus and theory of method. — A body of known moment of inertia is suspended by the sample of steel wire to be tested, thus constituting a torsion pendulum. Then

$$\frac{4\pi^2 K}{t^2} = b \qquad \text{(i)}\dagger$$

in which t is the time of one complete vibration of the torsion pendulum, b is the constant of torsion of the wire, and K is the moment of inertia of the suspended body.

The constant of torsion of a wire is given by the equation

$$b = \frac{\pi n R^4}{2L} \qquad \text{(ii)}\ddagger$$

in which n is the slide modulus of the material of the wire, R is the radius of the wire, and L is the length of the wire.

* See Franklin and MacNutt, *Elements of Mechanics*, pages 191 and 192.
† See Franklin and MacNutt, *Elements of Mechanics*, pages 137 to 140.
‡ See Franklin and MacNutt, *Elements of Mechanics*, pages 191 and 192.

By substituting the value of b from equation (i) in equation (ii) we have

$$n = \frac{8\pi KL}{R^4 t^2} \qquad \text{(iii)}$$

so that the value of the slide modulus n may be calculated when K, L, R, and t are known.

A circular disk of metal is used for the suspended body so that

$$K = \tfrac{1}{2} m r^2 \qquad \text{(iv)}$$

where m is the mass of the disk and r is its radius.

Units of the c.g.s. are supposed to be used in equations (i), (ii), (iii), and (iv). If units of the foot-pound-second system are used the value of n as given by equation (iii) will be in poundals per square inch which result must be divided by 32.2 to reduce the value of n to pounds per square inch.

Work to be done.— Weigh the disk and measure its diameter. Suspend the disk, measure the length and diameter of the wire, and determine the vibration period of the disk when it vibrates about the wire as an axis. The vibration period may be determined with sufficient accuracy by observing the time required for a number of complete vibrations, say twenty or more.

The greatest source of error is in the measurement of the diameter of the sample of wire, and therefore the diameter should be measured at a number of places so as to give a fairly accurate mean result.

Computations and results.— Calculate the moment of inertia of the disk using equation (iv), calculate the constant of torsion of the wire using equation (ii), and calculate the slide modulus of the material of the wire using equation (iii).

EXPERIMENT 40.

Calibration of a Pressure Gauge.

The object of this experiment is to calibrate a pressure gauge and plot a curve of which the abscissas represent gauge readings,

and the corresponding ordinates represent the true values of the pressure indicated by the gauge.

Apparatus. — The gauge tester affords the most convenient means for calibrating a pressure gauge. The gauge tester consists of a chamber filled with oil into which a small plunger of known area *a* is forced, as shown in Fig. 54. The force acting on the plunger is equal to the weight upon the pan (including the weight of piston and pan), and the pressure produced at the lower end of the piston is equal to the quotient obtained by dividing this force by the area of the end of the plunger. Known pressures thus produced are applied to the gauge to be tested and the corresponding readings of the gauge are taken.

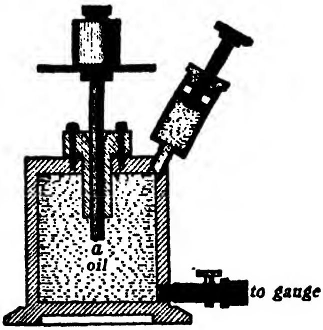


Fig. 54.

The gauge tester is provided with an auxiliary piston by means of which additional oil may be forced into the chamber so as to keep the plunger and pan floating.

Work to be done. — Connect the gauge to the tester, adjust the auxiliary piston until the plunger *a* floats, and note the reading of the gauge when no weights are on the pan.

Place a series of increasing weights upon the pan, recording the value of the weights used in each case, and taking the corresponding gauge readings. Then unload the pan step by step taking a series of readings as before. Repeat both series of readings twice, at least.

Before each reading make sure that the plunger *a* floats and give the pan a slight turning motion to eliminate friction.

Remove the plunger and pan and measure the diameter of the plunger and weigh the plunger and pan. (It may be desirable to ask the instructor concerning the weights of the plunger and pan and the diameter of the plunger in order to avoid the damage that would result if the plunger were scratched.)

Computations and results. — From the data obtained above, calculate the value of the various pressures applied to the gauge, and plot the calibration curve of the gauge using gauge readings as abscissas and true pressures as ordinates.

Note. — The pressure produced by the gauge tester when calculated as explained above is the pressure at the end of the plunger. With increasing weight on the pan the column of oil rises in the tube which leads to the gauge because of the compression of the air in the upper part of the tube, and therefore the pressure in the gauge differs slightly from the pressure at the end of the plunger on account of the varying height of the column of oil in the connecting pipe. The error thus produced is never more than a small fraction of a pound per square inch.

EXPERIMENT 41.

Measurement of Altitude by the Barometer.

The object of this experiment is to determine the difference in level of two stations by observing the barometer readings b_1 and b_2 at the respective stations.

Work to be done. — When the stations are at a great distance apart it is necessary to use two barometers and two observers so that simultaneous barometer readings may be taken at the two stations. If the two stations are near together the observations may be taken by one observer as follows: Set up the barometer at station A and take the reading of the barometer and of the attached thermometer, then carry the barometer to station B and take the readings of the barometer and of the attached thermometer, and then carry the barometer back to station A and take readings of the barometer and of the attached thermometer. The mean of the two sets of readings at station A may be assumed to be simultaneous with the readings obtained at station B.

To carry a barometer, first turn the screw S, Fig. 12, until the mercury is nearly to the top of the barometer tube, then

bring the barometer carefully to a horizontal position in unhooking it from its support, and carry the instrument up-side-down. To set the barometer up bring it to a horizontal position, and in hanging it up bring it carefully to a vertical position, and then uncrew S (not too far!).

Computations and results. — Let b_1 and b_2 be the corrected barometer readings (see Experiment 3) and let t_1 and t_2 be the temperatures of the air (and barometer) at the two stations. The difference $b_1 - b_2$ is the height of mercury column required to balance a column of air whose height H is equal to the difference in level of the two stations. Therefore, if d is the density of the mercury (at zero, of course, inasmuch as the barometer readings have been reduced to zero) and δ the mean density of the air between the two stations, then

$$H = \frac{d(b_1 - b_2)}{\delta} \tag{i}$$

inasmuch as the heights $(b_1 - b_2)$ and H of the balanced columns are inversely proportional to the respective densities of mercury and air.

When the stations are not too far apart the mean pressure of the intermediate air is approximately $\frac{1}{2}(b_1 + b_2)$ and its temperature is approximately $\frac{1}{2}(t_1 + t_2)$, so that

$$\delta = 0.001293 \times \frac{b_1 + b_2}{2 \times 760} \times \frac{273}{273 + T} \tag{ii}$$

in which 0.001293 is the density of dry air at 0° C. and standard atmospheric pressure, and T is written for $\frac{1}{2}(t_1 + t_2)$.

Substituting the value of δ from equation (ii), and using 13.6 for d, we have

$$H = 16000 \times \frac{b_1 - b_2}{b_1 + b_2} \times \frac{273 + T}{273} \tag{iii}$$

which expresses the difference of level between the two stations *in meters* with a sufficient degree of accuracy for most purposes.*

* For a more accurate formula see Kohlrausch, *Physical Measurements*, page 78.

EXPERIMENT 42.

ADJUSTMENT AND TESTING OF A SPIRIT LEVEL.

The object of this experiment is to adjust a spirit level and to determine the angular movement of the level corresponding to one division of movement of the bubble.

Adjustment of the level. — Figure 55 shows a simple form of spirit level with screws at the ends of the level tube by means of which the axis of the tube may be so adjusted with respect to the base plate that the base plate is level when the bubble stands in the middle of the tube. The knurled head at the middle serves as a handle for lifting the level. When the level is in proper adjustment it gives the same indication whether it be in a given position or turned end for end.

Fig. 55.

The ideal method of adjusting a level would be to place the instrument upon an accurately horizontal surface and adjust the tube until the ends of the bubble are at equal distances from the center of graduations. This procedure, however, is impracticable because accurately an horizontal surface is seldom available. The practical method for testing the level is to place it upon a beam and adjust the beam until the bubble is at the center of the tube; then reverse the level and bring the bubble to the center of the tube, *half* by adjusting the beam and *half* by adjusting the level tube. Reverse the level and repeat this operation until the bubble stands in the center of the tube for both positions of the level. The level is then in adjustment and the beam is horizontal.

Determination of sensitiveness. — Figure 56 shows a beam carrying two Y-supports in which the level tube to be tested is placed, and at one end of the beam is a micrometer screw by means of which the inclination of the beam may be altered by known amounts. Such a device is called a *level tester*. The angular movement of the beam for each turn of the micrometer

screw is easily determined from the length of the beam (measured from fulcrum to the tip of the screw) and the pitch of the screw.

The sensitiveness of the level is determined by finding the angular movement of the beam which causes the bubble in the level tube to move one division.

Work to be done. — (*a*) Adjust a spirit level, following the method described above.

(*b*) Place the level tester upon a hard smooth surface, place the level tube upon the Y-supports, turn the micrometer screw

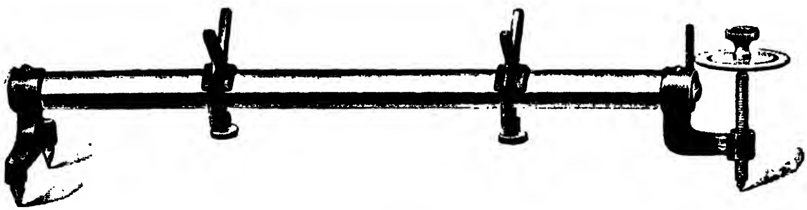


Fig. 56.

until one end of the bubble is at the extreme end of the divided scale on the level tube, and record the reading of the micrometer screw and the readings of both ends of the bubble on the scale which is etched upon the tube. Then turn the screw step by step taking simultaneous readings of micrometer screw and ends of bubble until the bubble has reached the other end of the level tube. Repeat this set of observations at least four times.

Measure the length of the beam from the fulcrum to the tip of the micrometer screw, determine the pitch of the screw (see instructor), and measure the length of the graduations on the level tube.

Computations and results. — Plot a curve of which a given ordinate represents the mean of all the readings of both ends of the bubble for a given reading of the micrometer screw and of which the corresponding abscissa represents the angular movement of the beam of the tester reckoned from some chosen zero position. The most convenient zero position is the initial position of the beam of the level tester in the above observations. From

a set of bubble readings corresponding to a given reading of the micrometer screw, calculate the probable error of one setting of the level tube. Reduce this probable error to angle.

Specify the magnitude of an error in the measured length from fulcrum to tip of micrometer screw which would produce in the calculated value of the angle corresponding to one division of movement of the bubble, an error equal to, say, one fourth of the probable error of one setting.

The inside surface of the level tube against which the bubble rests has the form of a cylinder bent into the arc of a circle. Find from the data obtained under (*b*) above the radius of this circle.

EXPERIMENT 43.

A Study of the Compressibility of Air.

The object of this experiment is to investigate the relation between pressure and volume of a given amount of air at a constant temperature.

Apparatus. — The most convenient form of apparatus for this experiment is shown in Fig. 57, the apparatus being fixed permanently to the wall of the laboratory. An iron pipe *I*, communicates at its lower end with an open manometer tube *m* and with a closed tube *t* in which a small amount of air is entrapped. A plunger *P* may be adjusted up or down in the iron pipe *I* so as to bring the mercury to any desired level in the manometer tube *m*, and thus subject the entrapped air in *t* to any desired pressure. The pressure of the entrapped air is $b \pm h$, where *b* is the atmospheric pressure as indicated by a barometer, and *h* is the difference in level as shown in Fig. 57. The volume of the entrapped air is proportional to the length *v* in Fig. 57, and for the purpose of this experiment this length may be taken as a measure of the volume of the entrapped air.

A simpler form of apparatus, but one which requires great care to avoid the spilling of the mercury is shown in Fig. 58. The manometer tube *m* is connected with the tube *t* in which the

air is entrapped by means of a rubber tube, and the variations of pressure are obtained by moving the tube m up and down.

Work to be done. — Adjust the apparatus so as to obtain in succession six different values of h, beginning with the greatest

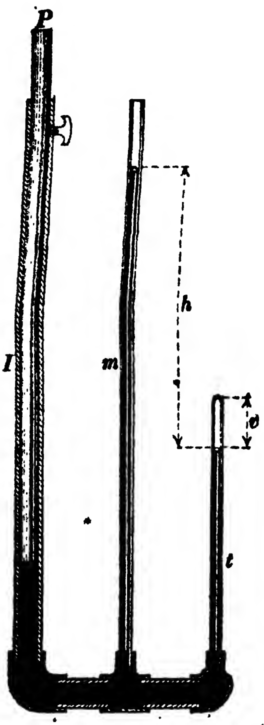


Fig. 57.

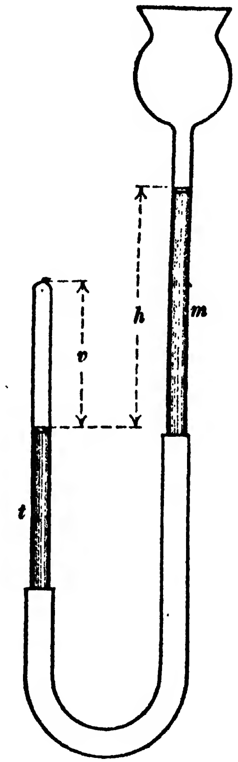


Fig. 58.

possible value and ending with the least possible value, and after each adjustment tap the tubes gently so as to bring the mercury meniscus in each tube up to its normal shape, and measure the values of h and v carefully by means of a cathetometer or reading telescope.

Read the barometer * at the beginning and at the end of the

* There is no need for the purposes of this experiment of correcting the barometer readings for systematic errors.

experiments and use the mean of these two barometer readings for b above.

In carrying out these observations care should be taken to avoid temperature changes especially of the small tube which contains the entrapped air.

Computations and results. — Tabulate the pairs of values found for v and $b \pm h$* from the above observations. According to Boyle's law, the product of the pressure and the volume of a given amount of gas is equal to a constant when the temperature does not change. Tabulate, therefore, the values of $v(b \pm h)$ along with the values of v and $b \pm h$. Determine the mean of the tabulated values of $v(b \pm h)$, and tabulate the differences between this mean and each of the tabulated values of $v(b \pm h)$. These differences represent errors of observations.†

Plot the pairs of values of v and $b \pm h$, using values of v as abscissas and values of $b \pm h$ as ordinates, and draw a smooth curve among the points so plotted.

EXPERIMENT 44.

A Study of Sliding Friction.

The object of this experiment is to investigate the approximate laws of sliding friction, and to determine the coefficient of friction between given sliding surfaces.

Theory. — When one body slides on another, a force opposing the motion is brought into existence. Let N be the normal force pushing two sliding surfaces against each other, and let T be the tangential force (that is the force parallel to the sliding surfaces) necessary to produce sliding. The relation between N and T is, in general, extremely complicated, and it depends

* The positive or negative sign is to be used in this expression according as the pressure of the entrapped air is greater or less than the pressure of the outside air.

† These differences are, in fact, partly due to inexactness of Boyle's law, the product of pressure times volume of a given amount of gas at a constant temperature is not a constant; but the inexactness of Boyle's law cannot be detected by rough measurements of the kind involved in this experiment.

more or less upon the area of the surfaces in contact and upon the velocity of sliding. When, however, the surfaces are comparatively smooth and when the sliding substances are not of the same material, the following relations are approximately true.

(*a*) The force T necessary to start the sliding is greater than the force required to maintain the sliding, value of N being given.

(*b*) The force T required to maintain the sliding is nearly independent of the velocity.

(*c*) The force T necessary to maintain the sliding is nearly independent of the area of the rubbing surfaces, N being given.

(*d*) The force T required to maintain the sliding is approximately proportional to N. That is, for given materials and given character of rubbing surfaces, the ratio T/N is approximately constant. The ratio is called the *coefficient of friction* μ of the given surfaces.

Work to be done.* — Pin a smooth piece of paper upon a board provided for the purpose, and place the board in a horizontal position. Weigh the wooden block to be used, and add its weight to each of the weights used in the following work. Using a succession of values of weight placed on the block, determine, by means of a spring dynamometer, the force required to *start* the block, the force required to *move the block uniformly* at a small velocity, and the force required to *move the block uniformly* at a greater velocity.

From the data obtained compute the mean coefficient of starting friction, and the mean coefficient of sliding friction for each of the two velocities used, and plot curves showing the relation of T to N for each of the three cases.

EXPERIMENT 45.

SURFACE TENSION OF WATER.

The object of this experiment is to determine the surface tension of pure water.

Theory. — *Method* (*a*). When the end of a capillary tube is

* Adapted from Hall and Bergen.

dipped into clean water the water rises in the tube to a certain definite height above the surface of the water in the vessel. The column of water thus formed is supported by the tension of the surface of the water in the tube, and inasmuch as the surface of the water in the tube is tangent to the walls of the tube (if the walls are clean) it is evident that *the tension in the breadth* $2\pi r$ (equal to the circumference of the bore) *of water surface is equal to the weight of the suspended water column* so that

$$2\pi r T = \pi r^2 h d g$$

so that

$$T = \frac{hdgr}{2} \tag{i}$$

where T is the tension of unit width of water surface, h is the height to which the water rises in the capillary tube, d is the density of the water, g is the acceleration of gravity, and r is the radius of the bore of the tube. Units of the c.g.s. system are understood to be used throughout.

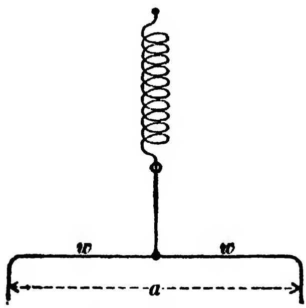


Fig. 59.

Method (*b*). A clean piece of wire ww, Fig. 59, with down turned ends, is suspended from the spring of a Jolly balance, and the force F required to pull the wire out of a tumbler of perfectly clean water is observed. Then

$$T = \frac{F}{2a} \tag{ii}$$

where a is the dimension shown in Fig. 59, and the factor 2 is introduced because the film which is pulled up by the wire has two surfaces.

In order to determine the value of the force F from the observed elongation of the spring, it is necessary to standardize the spring by observing the elongation produced by a known weight.

Work to be done —*Method* (*a*). The capillary tube, the beaker containing the water, the thermometer to be used for reading the temperature, and the glass scale to be used in measuring h must all be carefully cleaned with nitric acid, then with caustic potash, and then thoroughly rinsed with clean water Great care must be taken to keep the surface of the water free from dust and from every trace of oil. Before handling any of the apparatus the hands should be washed with soap and thoroughly rinsed in clean water.

Place the glass scale in a vertical position with its lower end dipping into the water in the beaker. Dip the capillary tube deep into the water and then raise it a little in order that the inner surface above the meniscus may be wet, and clamp it in a vertical position in front of the scale. Read the position of the meniscus in the tube, taking care to avoid parallax error by holding the eye on a level with the meniscus. Read the position of the surface of the water in the beaker by sighting along the surface underneath. Observe the temperature of the water.

Repeat these observations several times.

Measure the bore of the tube by means of the micrometer microscope as explained in Experiment 9. If a micrometer microscope is not available the bore of the tube may be determined as follows: Dry the tube, fill a portion of it with mercury, measure the length of the mercury thread, then allow the mercury to flow out without loss into a small dish and weigh. The temperature of the mercury in the tube is to be observed. From these data the radius of the tube may be calculated.

Method (*b*). Standardize the spring of a Jolly balance by observing the elongation of the spring produced by a one-gram weight. Suspend the wire *ww*, Fig. 59, from the spring and note the reading of the balance. Then bring a tumbler of water, perfectly clean, up from below so as to submerge the wire, and then observe the maximum elongation of the spring when the tumbler of water is lowered. Measure the dimension a, Fig. 59.

Computations and results. — From the data obtained, calculate

the surface tension of water by equation (i) and by equation (ii) and compare the results.

EXPERIMENT 46.

Coefficient of Viscosity of a Liquid.

The object of this experiment is to determine the coefficient of viscosity of a liquid.

Theory.—*Definition of coefficient of viscosity.* Consider a thin layer of fluid of thickness x lying between two flat plates AA and BB as shown in Fig. 60, and suppose that the plate AA is

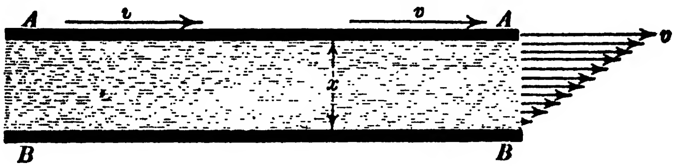


Fig. 60.

moving at velocity v as indicated by the arrows. If the fluid between the plates were a viscous liquid like syrup, it is evident that a very considerable force would have to be exerted upon the plate AA to keep it in motion; in fact, any fluid whatever, whether liquid or gas, is more or less like syrup in this respect, and the force F with which the motion of the plate is opposed by the fluid is proportional to its area a, to its velocity v, and inversely proportional to the distance x between the plates; that is,

$$F = \frac{\eta a v}{x}$$

in which η is a proportionality factor which has a definite value for a given fluid and which is called the coefficient of viscosity of the fluid.

The flow of a liquid through a small smooth-bore tube is opposed by friction due to viscosity, and it can be shown that

$$V = \frac{\pi p R^4 t}{8 l \eta} \qquad \text{(i)*}$$

* See Franklin and MacNutt, *Elements of Mechanics*, page 243.

in which V is the volume of liquid discharged by the tube in t seconds, p is the pressure which is forcing the liquid through the tube, R is the radius of the bore of the tube, l is the length of the tube, and η is the coefficient of viscosity of the liquid. Therefore the value of η may be calculated when the amount of discharge V during a given time t has been observed, p, R, and L being known.

Apparatus. — A fine bore glass tube TT is arranged as shown

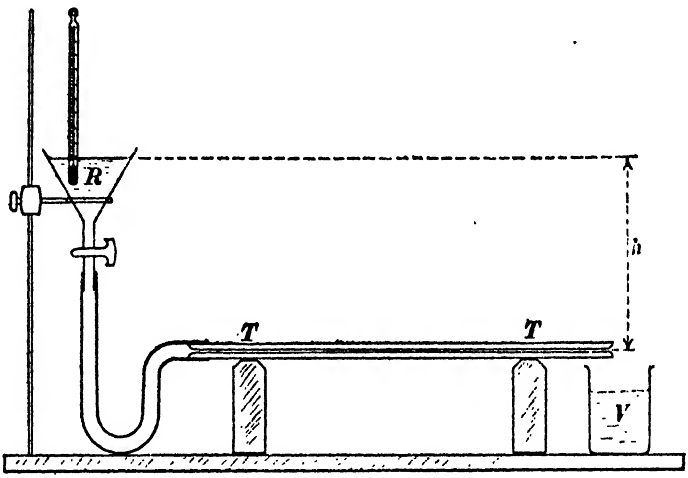


Fig. 61.

in Fig. 61 so as to discharge water from a reservoir R into a measuring vessel V.

Work to be done. — Measure the bore of the tube TT, preferably by the method of weighing as explained in Experiment 44, and measure its length l. Place the tube in position, observe the temperature of the water, collect the discharge in the measuring vessel for an observed interval of time t, and read off the volume V of discharge. Observe the height h, Fig. 61, immediately before and immediately after collecting the discharge V.

Repeat these observations for a series of values of h, taking several sets of observations for each value of h. Refill the reser-

voir after each discharge so as to keep the value of h nearly invariable.

Computations and results. — Average the values of h and the values of V for each set of observations as taken above, and plot a curve of which the abscissas represent these average values of h and the ordinates represent the corresponding average values of V. This curve would be a straight line passing through the origin of coördinates if equation (i) were exactly true.* Therefore, draw a straight line through the origin of coördinates and among the plotted points so as to come as near to all the plotted points as possible, and the best data from which to determine the coefficient of viscosity of the liquid are the abscissa (h) and ordinate (V) of any chosen point on this straight line. From this pair of values of h and V, calculate the value of η from equation (i). For this purpose the pressure p which is forcing the liquid through the tube is to be determined from h, by the equation

$$p = hdg$$

where h is expressed in centimeters, d is the density of the liquid in grams per cubic centimeter, and g is the acceleration of gravity in centimeters per second per second.

Note. — All quantities in the above discussion are understood to be expressed in the units of the c.g.s. system.

EXPERIMENT 47.

The Venturi Water Meter.

The object of this experiment is to measure the rate of flow of water through a pipe by means of the Venturi water meter, and to check the readings of the meter by measuring the volume of discharge during a given time.

Apparatus. — The *Venturi tube* consists of a short piece of pipe

* Any systematic deviation of the plotted points from a straight line indicates either a systematic source of error or an inaccuracy in equation (i). In fact, equation (i) is not strictly true.

with a smooth bore and having a contracted place or throat as shown in Fig. 62. In this figure a' represents the sectional area of the bore at A, p' represents the pressure of the liquid at A, and v' represents the velocity of the liquid at A; and a'', p'',

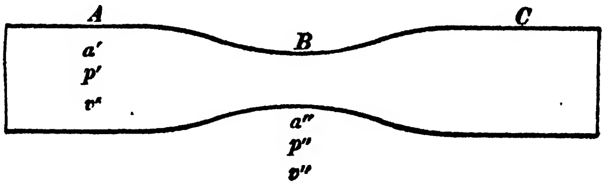


Fig. 62.

and v'' represent the corresponding quantities at B. The bore at C is supposed to be the same in diameter as at A.

When an approximately frictionless and incompressible fluid flows through the Venturi tube, the pressure at B is less than at A or C, in fact, the difference in pressure at A and B is given by the equation

$$p' - p'' = \tfrac{1}{2}\left(\frac{a'^2 - a''^2}{a''^2}\right) dv'^2 \qquad \text{(i)*}$$

in which everything is understood to be expressed in c.g.s. units, d being the density of the fluid.

The product $a'v'(= a''v'')$ is the number of units of volume of liquid passing through the pipe per second. Representing this rate of flow by Q, we have from equation (i):

$$Q = \left(\frac{2a'^2a''^2}{d(a'^2 - a''^2)}\right)^{\frac{1}{2}} (p' - p'')^{\frac{1}{2}} \qquad \text{(ii)}$$

Therefore if $p' - p''$ is measured, the value of Q (discharge of liquid per unit of time) may be calculated, inasmuch as a', a'', and d are constants for a given tube and for a given liquid.

Apparatus. — The Venturi water meter consists of a Venturi tube equipped with a device for measuring the value of $p' - p''$.

* See Franklin and MacNutt, *Elements of Mechanics*, page 230.

The Venturi meter to be used in this experiment is the commercial form of meter.* The essential features of this meter are shown in Fig. 63. The water to be measured flows through the

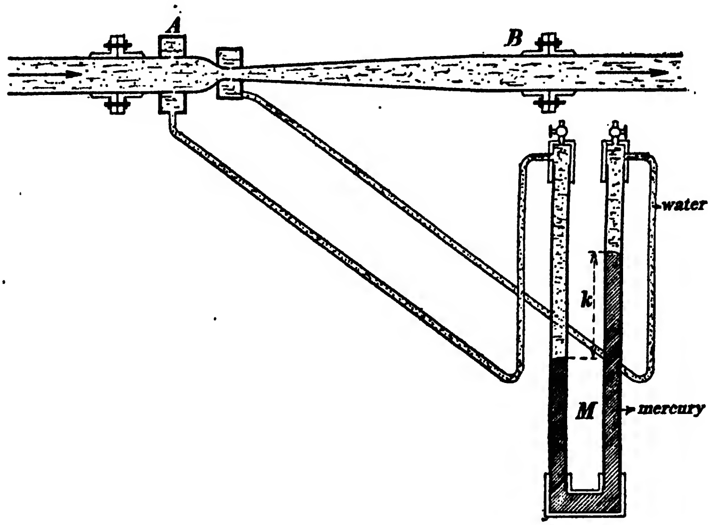


Fig. 63.

Venturi tube AB and the manometer M is arranged to indicate the value of $p' - p''$ as shown. The scale which is used for measuring the difference of level k is usually graduated to read gallons per minute directly; it is here assumed, however, that this scale reads in centimeters. To determine the value of $p' - p''$ in c.g.s. units (dynes per square centimeter) the value of k in centimeters is to be multiplied by the difference in density between water and mercury and by the acceleration of gravity.

The meter is arranged to discharge water into a cylindrical tank, and the volume of water discharged in a given time is to be determined by observing the rise of the water level in the tank during that time.

Work to be done. — Start the water flowing through the meter tube, close the discharge valve of the water tank, and observe

* Manufactured by the Builders' Iron Foundry, of Providence, R. I.

the clock reading of the instants when the water level in the tank reaches each of two marked points on a tall gauge-glass attached at one side of the tank.

Repeat this observation for a series of five rates of discharge ranging from a very low rate up to the highest rate for which measurements can be taken.

Repeat this whole set of observations, getting duplicate sets of data for each rate of discharge.

Measure the diameter of the tank and the distance between the marked points on the gauge-glass.

Find from the instructor the data concerning the diameter of the Venturi tube at the two points p and q, Fig. 63.

Computations and results. — Calculate the rate of discharge Q from each observed value of $p' - p''$. Represent these values of Q by Q'. Calculate the value of Q for each observed value of $p' - p''$ from the observed volumes of discharge. Represent these values of Q by Q''. Plot a curve of which the abscissas represent the values of Q' and the ordinates represent the values of Q''.

EXPERIMENT 48.

Relative Densities of Gases by Efflux.

The object of this experiment is to determine the ratio of the densities of several gases by the method of efflux.

Theory. — The volume of efflux V of an ideal incompressible frictionless fluid from a region under pressure p into a region under pressure p' is given by the equation

$$V = at\sqrt{\frac{2(p - p')}{d}} \qquad \text{(i)}$$

in which a is the area of orifice, t is the time of efflux, and d is the density of the fluid. Let t' be the time required for the same volume V of another ideal incompressible frictionless fluid of density d' to be discharged through the same orifice, the value of $p - p'$ being the same. Then

$$V = at'\sqrt{\frac{2(p-p')}{d'}}. \qquad \text{(ii)}$$

whence, dividing equation (i) by equation (ii), member by member, we find

$$\frac{d}{d'} = \frac{t^2}{t'^2} \qquad \text{(iii)}$$

That is, the ratio of the densities of the two fluids is equal to the ratio of the squares of the times of efflux.

Equation (iii) is not strictly accurate when applied to a gas because of the effects of friction and because of the effects of compressibility.* These effects, however, are usually less than one or two per cent. and they are ignored in the application of equation (iii) to the determination of the ratio of the densities of gases.

Apparatus.—A vessel VV, Fig. 64, is filled with a gas through the tube g, causing the mercury or oil to be displaced into the vessel M. The stop-cock C is then closed, the cock C' is opened, the gas escapes through a very fine orifice O and the clock reading is taken at the instant when the level of the mercury or oil reaches the point p and again at the instant when the level of the mercury or oil reaches the point p'. The difference of these two clock readings is the time required for the efflux of a definite volume of gas from the orifice O.

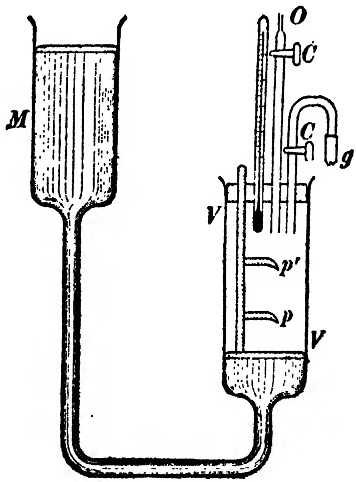


Fig. 64.

Work to be done. — (*a*) Fill the vessel V with dry hydrogen and empty it several times so as to wash out every trace of any residual gas which may remain in the vessel. Then observe the

* The effects of compressibility are to a large extent without influence on the result of this experiment because of the equal compressibility of the various gases to be tested.

time of efflux as above explained, and take the reading of the thermometer. This observation should be repeated several times.

(*b*) Fill the vessel *VV* with dry carbon dioxide in the same way and take a second set of observations of times of efflux and of temperature.

(*c*) Fill the vessel *VV* with dry air in the same way and take a third set of observations of times of efflux.

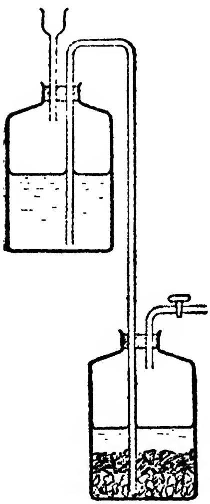


Fig. 65.

In order to dry the gases each gas should be passed through a calcium chloride tube. A hydrogen generator should be permanently set up and connected to a drying tube, a carbon dioxide generator should be permanently set up and connected to a second drying tube, and a small pump should be arranged for forcing air into the vessel *VV* through a third drying tube. In this way the above observations may be taken in a comparatively short length of time, the rubber tube *g*, Fig. 64, being attached to the one or the other of the drying tubes at will.

A convenient form of hydrogen generator is shown in Fig. 65. The lower bottle has a layer of pebbles in the bottom, and upon this layer of pebbles scraps of metallic zinc are placed (scraps of marble for the carbon dioxide generator), and the upper bottle is filled with dilute sulphuric acid (dilute hydrochloric acid for the carbon dioxide generator). The difference in level between the two bottles in Fig. 65 must be sufficiently great to generate the pressure necessary to force the gas into the vessel *VV*, Fig. 64.

PART III.

EXPERIMENTS IN HEAT.

LIST OF EXPERIMENTS.

49. Coefficient of linear expansion of a metal rod.
50. Coefficient of cubic expansion of glass.
51. Expansion of gases. The air thermometer.
52. Standardization of a mercury-in-glass thermometer.
53. Boiling points of a salt solution.
54. Curve of cooling.
55. Specific heat by water calorimeter.
56. Latent heat of fusion of ice.
57. Heat of combustion.
58. Ratio of specific heats of air.
59. Determination of vapor-pressure curve.
60. Hygrometry.
61. Flash test of kerosene.

THE USE OF THE THERMOMETER.

A well-made thermometer is an instrument of precision and it should be handled with care.

When a thermometer is heated beyond its range, the expanding mercury fills the bulb and stem, and the bulb bursts. This accident most frequently happens by carelessly allowing the bulb of a thermometer which reads barely up to 100°C. to come into contact with the bottom of a vessel in which water is being heated.

Always submerge as large a portion of a thermometer as possible in the bath of which the temperature is to be determined, leaving only enough of the thermometer projecting to enable the readings to be taken. Never remove the thermometer from the bath to take its reading.*

In reading a thermometer care should be taken to avoid the errors of parallax because of the very considerable distance between the thermometer scale and the mercury column. Errors of parallax are especially difficult to avoid when the thermometer reading is taked by means of a magnifying glass.

The best method of reading a thermometer accurately is by means of a reading telescope. Arrange the telescope so that its axis extended meets the thermometer scale at or near the end of the mercury column (end of mercury column in center of field of view of telescope), and arrange the thermometer so that the stem is as nearly as may be at right angles to the axis of the telescope.

The mercury column of a thermometer has a tendency to move by fits and starts especially when the temperature is falling. The thermometer should always be gently tapped on the end with a bit of soft wood before a reading is taken.

* This precaution may seem childish, but the authors have seen advanced technical students perform this ridiculous operation in the making of engineering tests.

The readings of a well-made chemical thermometer with a scale ranging from 0° C. to 100° C. may usually be relied upon to a fraction of a degree. Errors of 0°.2 are unusual. These remarks apply to the grade of chemical thermometer which sells for about $1.00.

Thermometers ranging to 200° C. or 300° C. frequently have errors of several degrees in the upper part of the scale.

For accurate work a thermometer must be standardized and calibrated as described in Experiment 52, and especial attention must be paid to the following sources of error:

(*a*) *Errors due to coolness of stem.* — The entire thermometer, bulb and stem, should be at the temperature which is to be indicated by the instrument. In the work outlined below this condition may be very nearly satisfied and no correction for coolness of stem need be made.*

(*b*) *Errors due to pressure.* — A thermometer is usually standardized with its stem vertical and it should be used in this position. When the stem is inclined or horizontal the pressure in the bulb due to the mercury in the stem is lessened, the bulb contracts slightly, and the thermometer reads too high. Changes of atmospheric pressure, or the pressure due to a liquid in which the thermometer is placed also cause changes of volume of the bulb. The errors due to variations of pressure seldom amount to more than 0.002 degree.

(*c*) *Secular shifting of readings.* — The reading of a thermometer corresponding to a given temperature is called simply a reading for brevity. After the bulb of a thermometer is blown it continues for a long time to contract irrespective of temperature changes. This contraction, which may continue to a sensible extent for a year or more, causes all of the readings of the thermometer to increase. This secular increase of readings may amount to one degree or even more if the thermometer is finished while the bulb is new. Good thermometers are kept for a long time before the graduations are made. This aging process is

* See Kohlrausch, London, 1894, page 85.

greatly accelerated by keeping the thermometer warm, say at 100° C.

(*d*) *Temporary shifting of readings.* — When the temperature of a thermometer is changed the corresponding change of volume of the bulb takes place only after the lapse of time. At high temperatures, such as 100° C. or higher, the change of volume corresponding to a given change of temperature takes place in a short time while at low temperatures, such as ordinary room temperature or lower, a very long time is required.

When a thermometer is kept for a long time at a given temperature the mercury settles to a definite reading called the "*stable reading*" corresponding to the given temperature. If the thermometer is heated and then cooled to a given temperature the thermometer reading will be below the stable reading corresponding to the given temperature, but it will rise towards its stable position, rapidly at first and then more and more slowly. Sometimes hours or even days elapse, before this temporary depression of a reading is obliterated. Any thermometer reading is temporarily lowered when the thermometer has been immediately before at a higher temperature than that corresponding to the reading, or temporarily raised when the thermometer has been immediately before at a temperature lower than that corresponding to the reading.

When changes of temperature do not exceed 20° C. this temporary shift of readings never exceeds one or two hundredths of a degree.

In precision thermometry the thermometer must be kept a long time at the temperature to be indicated or, where this is not feasible, a definite procedure must be followed. The same procedure may then be repeated at leisure and the temporary shift of the reading under the given procedure determined.

Example. — A thermometer, kept a long time at room temperature, is used to follow the comparatively rapid rise of temperature of a calorimeter to, say, 80° C. when the precise reading a is taken. The same thermometer after being again allowed to

stand for a long time at room temperature is again brought rapidly up to about 80°, plunged into a bath at about this temperature and its reading at once compared with the reading of another thermometer which has been for a long time at or near 80°. The difference between these two readings is the temporary shift of the reading *a* and is to be subtracted from the reading *a*. A more complicated and less satisfactory procedure is the one recommended by Pernet.*

EXPERIMENT 49.

Coefficient of Linear Expansion of a Metal Rod.

The object of this experiment is to determine the mean coefficient of linear expansion of a metal rod between ordinary room temperature and a temperature of approximately 100° C.

Apparatus. — The metal rod to be tested is surrounded by a steam jacket into the sides of which several thermometers are placed as shown in Fig. 66. Tap water entering at *e* may be

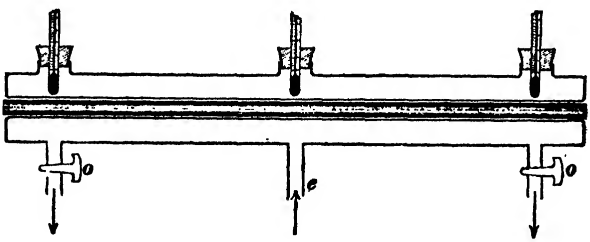


Fig. 66.

allowed to flow through the jacket passing out at *oo*, or steam may be allowed to enter the jacket at *e*, passing out at *oo*. The rod to be tested is but very slightly longer than the steam jacket, and it should lie in a closely fitting tube which passes through the center of the steam jacket, thus permitting of the use of any one of several rods.

In order to determine the coefficient of expansion of a rod accurately the total length of the rod need not be measured

* Winkelmann, *Handbuch der Physik*, Vol. II., part 2.

with great precision, *but the increase of length must be determined with great care.* Therefore, the total length of the rod may be measured by means of the ordinary meter scale.

To determine the increase of length a special micrometer caliper shown in Fig. 67 should be employed. The portion *ab* of the

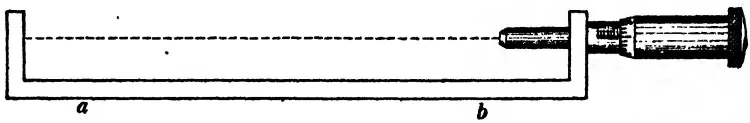


Fig. 67.

caliper should be covered with wood or other insulating material and the caliper should be applied to the rod as quickly as possible so as to avoid changes of temperature of the caliper. The increase of length of the rod may then be determined by taking the caliper reading when the rod is at the lower temperature and again when the rod is at the higher temperature.

Work to be done. — Place the rod to be tested in the steam jacket, pass a stream of water through the jacket until the entire apparatus settles to a uniform temperature t, and then observe the thermometer readings, and take the reading of the special caliper as quickly as possible.

Then pass steam through the jacket until the whole reaches a uniform temperature t', read the thermometers and again quickly take a reading of the special micrometer.

Measure the length of the rod at room temperature by means of an ordinary meter scale.

Particular care must be taken to avoid changes of temperature of the special micrometer during the progress of the above measurements.

Take a similar set of measurements on each of the rods to be tested.

Computations and results. — Calculate the mean coefficient of expansion α on each rod from the formula

$$\frac{L}{L+l} = \frac{1 + \alpha t}{1 + \alpha t'} \tag{1}$$

in which L is the length of the rod at temperature t,* and l is the increase of length when the temperature is raised from t to t'.

EXPERIMENT 50.

Cubic Expansion of a Glass Vessel.

The object of this experiment is to determine the mean coefficient of cubic expansion of the glass of a specific gravity bottle.

Method. — The volume V (cubic contents) of the bottle at temperature t and the volume V' (cubic contents) of the bottle at temperature t' may be determined by the method explained in Experiment 18, and the mean coefficient of cubic expansion β of the glass may be calculated from the equation

$$\frac{V}{V'} = \frac{1 + \beta t}{1 + \beta t'} \tag{i}$$

Work to be done. — Determine the cubic contents V of the bottle at a temperature t which is very slightly above room temperature, as explained in Experiment 18.

Then place the bottle filled with water into a bath of boiling water (the bottle should not rest on the bottom of the vessel), stir the bath, and when the bottle has reached the temperature of the bath, observe the temperature of the bath with great care, wipe off the excess of water from the stopper of the bottle, remove the bottle from the bath, wipe it dry, allow it to cool, and weigh.

Considerable error may be made in determining the actual temperature of the bottle when it is in the bath. The essential conditions are that the bath be kept as nearly as possible at constant temperature while the bottle is in it and that ample time be allowed for the bottle to settle to the temperature of the bath.

The entire set of observations should be repeated several times.

Computations and results. — From the data above determined calculate the values of V and V' as explained in Experiment 18, and then calculate the mean coefficient of cubic expan-

* Or at room temperature.

sion of the glass between the two temperatures t and t', using equation (i).

EXPERIMENT 51.

THERMAL EXPANSION OF GASES. RATIO OF STEAM TEMPERATURE TO ICE TEMPERATURE.

The form of thermometer which has been adopted as the ultimate standard for measuring temperature is the hydrogen thermometer. A constant volume of hydrogen is brought in succession to the two temperatures T' and T'' which are being measured, and the corresponding pressures of the hydrogen p' and p'' are observed. Then, according to the definition of temperatures as measured by the gas thermometer, we have

$$\frac{T'}{T''} = \frac{p'}{p''} \tag{i}$$

so that if one of the temperatures is known the other may be calculated. Temperatures measured in this way are called *absolute temperatures*. Before any temperature can be measured by the gas thermometer it is necessary to assign an arbitrary value to a given temperature, or to assign an arbitrary value to a given difference of temperature; the difference between steam temperature and ice temperature, under standard conditions as to pressure, is taken arbitrarily to be equal to 100 units or degrees. On the basis of this agreement, the first step in the use of the gas thermometer is to determine the actual value of steam temperature and of ice temperature, or the ratio of steam temperature to ice temperature as expressed in equation (i). The manipulation of the gas thermometer, however, is very difficult and therefore this form of thermometer is not at all suited to elementary laboratory work.

The object of this experiment is to determine the ratio of steam temperature to ice temperature by means of a very simple but inaccurate form of air thermometer.

Apparatus and work to be done. — A long glass tube, having a uniform bore one or two millimeters in diameter, has one of its

ends sealed. The tube is slightly warmed and its open end is dipped into mercury. As the tube cools, a small portion of the mercury is drawn into it, and this mercury globule is used in the following work as if it were a piston serving by its position to indicate the volume of the entrapped air.

(*a*) To bring the mercury globule to the desired position, insert a fine iron wire into the open end of the tube until it projects beyond the mercury globule. The entrapped air can then escape between the wire and the walls of the tube, and the globule may be allowed to move along the tube until it is not more than three-quarters of the distance from the closed end of the tube. This preliminary adjustment of the mercury globule is necessary in order that the volume of the entrapped air may not exceed the total volume of the tube at steam temperature.

(*b*) Place the tube horizontally in a trough filled with chopped ice, being careful to prevent moisture from entering the open end of the tube. After the tube has settled to ice temperature, measure the distance l' from the closed end of the tube to the mercury globule.

(*c*) Place the tube horizontally in a steam jacket as shown in Fig. 68, the open end of the tube projecting so as to prevent the

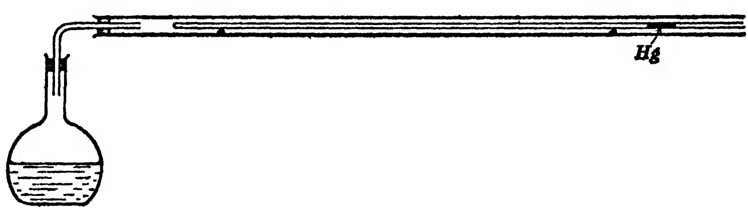


Fig. 68.

entrance of moisture. When the tube is settled to the temperature of the steam, measure the distance l'' from the closed end of the tube to the mercury globule.

Note. — The temperature of the steam will be less than standard steam temperature if the atmospheric pressure is less than 760 millimeters. Therefore it is desirable to place an accurate

thermometer in the steam jacket and take the reading of the actual steam temperature.

Computations and results. — The ratio of the two distances l' and l'', as measured above, is equal to the ratio of the volumes of the entrapped air at ice temperature and steam temperature respectively, and this ratio of the volumes of the entrapped air is equal by definition to the ratio of the two temperatures.

Calculate the ratio of ice temperature to steam temperature from the observed values of l' and l''.

If this result differs appreciably from the ratio 273:373 the discrepancy may be due in large part to non-uniformity of bore of the tube, it may be due in part to errors in measuring the distances l' and l'', and it may be due to the fact that the steam temperature is less than standard steam temperature, as explained above.

EXPERIMENT 52.

Standardization of a Mercury-in-Glass Thermometer.

The object of this experiment is to standardize a mercury-in-glass thermometer.

Theory. — The ideal mercury-in-glass thermometer has a stem of which the bore is perfectly uniform, its zero mark corresponds exactly with standard ice temperature, its 100° mark corresponds exactly with standard steam temperature, and the space between the zero and the 100° mark is divided into 100 exactly equal parts. Consider any two readings t and t' upon the scale of such a thermometer. Let v be the volume of the bore of the stem between ice point and the point t, and let v' be the volume of the bore of the stem between ice point and the point t'.

Then it is evident that

$$\frac{t}{t'} = \frac{v}{v'} \tag{i}$$

This equation constitutes, as it were, a definition of mercury-in-glass temperature.

In the ordinary thermometer the zero point and the 100° point

may be inaccurately located, and the bore of the stem may not be uniform. In the standardization of such a thermometer, therefore, it is necessary to test the thermometer at standard ice temperature I and at standard steam temperature S, and to calibrate the bore of the stem in order that its readings may be correct.

See page 140 for detailed directions as to the method of using a thermometer.

Work to be done. — (*a*) *Determination of ice point.* — Take some finely chopped clean ice, rinse it with a small amount of distilled water. Fill a small glass vessel completely full of this ice and pour in enough distilled water to barely cover the ice. Fix the thermometer to a clamp stand and sink it in the ice bath until the zero point is just visible above the surface of the bath. Take and record the reading of the thermometer at intervals until the reading has reached its lowest value. This lowest reading a is the required ice point.

The ice point must be taken before the steam point inasmuch as the ice point will be temporarily lowered after the thermometer has been in the steam bath. If the thermometer has been a long time at room temperature before the ice point is determined, the reading of the ice point will be very nearly the stable reading of the ice point.

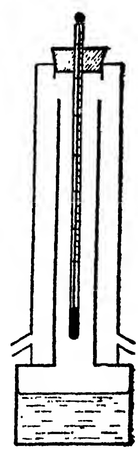


Fig. 69.

(*b*) *Determination of reading f corresponding to a known temperature t near the standard steam temperature.* — Place the thermometer in a jacketed steam bath as shown in Fig. 69, leaving only enough of the stem projecting to permit of readings being taken. Take and record the reading of the thermometer at intervals until the reading has reached its highest value. This highest reading f is the one to be used. The barometer reading b' is to be taken immediately. This reading b' is to be corrected for temperature and gravity. (See Experiment 3.) Let b be

the corrected barometer reading in millimeters. Then the true temperature corresponding to the reading f is:

$$t = 100^\circ - 0.0375(760 - b) \qquad \text{(ii)}$$

This equation results from the observed fact that the temperature of a steam bath in the neighborhood of 760 mm. pressure decreases 0.0375 degree Centigrade for each millimeter decrease of pressure.

Caution. — Take care that the thermometer bulb does not dip into the water. Otherwise the temperature will be above the standard value.

(*c*) *Calibration.* — Detach a portion of the mercury filament of the thermometer of a length very nearly equal to an aliquot part (one quarter, say) of the distance from 0° to 100°. This may be done as follows: Invert the thermometer, tap it slightly so as to detach a portion of the filament and allow this portion to move to the end of the stem. Then turn the thermometer right end up and observe the reading s on the scale, of the point where the mercury columns come together. Warm or cool the bulb until the mercury stands at reading $s + l$, where l is the desired length, quickly invert and tap the thermometer and the mercury column will separate at s, giving a detached portion of length l.

Place this detached filament with its lower end exactly coincident with the reading a and take the reading b of its upper end (the thermometer stem must of course lie in a horizontal position); shift the filament until its lower end is exactly coincident with reading b and take the reading c of its upper end; and so on. If the detached filament is nearly 25° in length the four readings b, c, d, and e will thus be obtained.

(*d*) *Depression of Ice Point.* — After the ice point has been determined under (*a*) above, place the thermometer for several minutes in boiling water, and again determine the ice point to determine the temporary lowering of the ice point.

Computations and results. — It is desired to calculate the true mercury-in-glass temperature corresponding to readings b, c, and d, and to plot a curve of which the abscissas represent the read-

ings a, b, c, d, and f and the ordinates the corresponding true temperatures I, B, C, D and t. From this curve one may find the true mercury-in-glass temperature corresponding to any given reading.

Assume the bore of the stem to be uniform throughout the section d to f, see Fig. 70, let V be the volume of the bore between a and f, and x the volume of the detached filament. Then

$$V = 4x + \left(\frac{f-e}{e-d}\right)x \qquad \text{(iii)}$$

for, since the bore is assumed uniform between d and f the volume of the portion ef is $\left(\frac{f-e}{e-d}\right)x$.

The true temperatures B, C and D may now be calculated with the help of equation (i). For example:

$$C = \left[\frac{2x}{4x + \left(\frac{f-e}{e-d}\right)x}\right] \times t = \frac{2}{4 + \left(\frac{f-e}{e-d}\right)} \times t$$

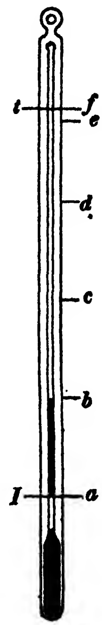


Fig. 70.

EXPERIMENT 53.

Boiling Points of Solutions.

The object of this experiment is to determine the relation between the boiling point and the concentration of a salt solution.

Apparatus and theory. — The boiling point of a solution is the temperature at which the solution and pure steam can stand together in equilibrium at standard atmospheric pressure. At any temperature greater than this the solution vaporizes, and at any temperature less than this, the steam condenses. It is rather difficult to determine the true boiling point of a solution because the solution itself when boiling is almost sure to be above its true boiling point, and because the vapor from the solution being in contact with drops of pure water on the walls of

the vessel is apt to cool down to the boiling point of pure water.

The boiling point of a salt solution may be determined with greatest accuracy by allowing steam (of course, pure steam) to bubble up through the salt solution. In this case the salt solution is heated up to its boiling point by the condensation of the steam.* Steam is generated in a flask *A* and allowed to bubble up through the salt solution in flask *B* in which is a thermometer for indicating the boiling point of the solution. Flask *B* may have a scale on its side so that the increasing volume of the solution can be read off, or it may be placed on a small platform scale so that the amount of condensed steam can be determined by weighing. In this way a concentrated solution of salt may be used at the start, and as the solution becomes more dilute, because of the addition of condensed water, its concentration may be determined at any instant.

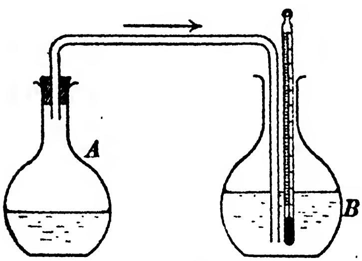


Fig. 71

Work to be done. — Set up the apparatus as shown in Fig. 71, using a saturated solution of common salt in flask *B*, and take a series of simultaneous readings of thermometer and volume or weight (mass) of salt solution. The actual amount of salt in the solution in grams should be determined at the start and the concentration at any stage in the experiment should be expressed as so many grams of salt per hundred grams of solution.

A thermometer reading to tenths of a degree should be used in this experiment inasmuch as the total range of boiling point of the salt solution will be only a few degrees.

Computations and results. — Plot a curve of which the abscissas represent concentrations of salt solution in grams of salt per

* This action is rather paradoxical inasmuch as the salt solution is heated to a higher temperature than the steam which heats it; this action is very much like the action of pure ice in cooling a salt solution to a lower temperature than that of the ice when it is put into the solution.

hundred grams of solution, and of which the ordinates represent the observed boiling points.

EXPERIMENT 54.

Curve of Cooling.

The object of this experiment is to determine the curve of cooling of a vessel of hot water, to determine the rate of cooling at several specified temperatures, and to compute the rate at which heat would have to be supplied to the vessel to hold the temperature at any prescribed value.

Apparatus and work to be done. — An ordinary tin can of about one quart capacity is suitable for this experiment. It should be provided with a stirrer of the same metal. Weigh the metal vessel and stirrer, fill it nearly full of water and weigh again. Heat the vessel and the water to the boiling point. Stand the vessel where it will be shielded from draughts of air, and place in it a thermometer. Place another thermometer in the open air of the room at a short distance from the vessel, and take readings of both thermometers every minute until the vessel is cooled nearly to room temperature. The water is to be stirred between readings.

Computations and results. — Plot the curve using times as abscissas and differences of readings of the two thermometers as ordinates, and from this curve determine the rate of cooling of the vessel at 20°, 40°, and 60° above room temperature (see page 13 of the introduction). Then calculate the rate at which heat would have to be supplied to the vessel to keep it at each specified temperature.

Let W be the weight (mass) of water in grams, m the weight (mass) of the metal vessel and stirrer in grams, and S the specific heat of the metal vessel. Then the vessel and stirrer are together equivalent to mS grams of water, and in cooling through Δt degrees, the vessel gives off an amount of heat ΔH which is given by the equation

$$\Delta H = (W + mS)\Delta t \tag{i}$$

Dividing both members of this equation by the interval of time ΔT during which the small drop of temperature takes place, we have

$$\frac{\Delta H}{\Delta T} = (W + mS)\frac{\Delta t}{\Delta T} \tag{ii}$$

but $\Delta t/\Delta T$ is the rate of cooling of the vessel and therefore $\Delta H/\Delta T$ is the rate at which heat is given off, which is equal to the rate at which heat would have to be supplied to the vessel to keep its temperature constant.

EXPERIMENT 55.

Specific Heat by the Water Calorimeter.

The object of this experiment is to determime the mean specific heat of a substance by means of the water calorimeter.

Theory. — A body of mass M is heated to temperature t, and plunged into a mass W of water at temperature t'; the two settle to thermal equilibrium at temperature t'', and the specific heat S of the body may be calculated from the equation

$$S = \frac{W(t'' - t')}{M(t - t'')} \tag{i}$$

This equation is true only when no heat is gained from or lost to surrounding objects. The containing vessel and the thermometer are warmed together with the water, and, therefore, their water equivalents must be added to the mass of the water to give the value of W in equation (i). Furthermore, precautions must be taken to prevent the exchange of heat between the calorimeter and its surroundings, or else the amount of such exchange must be determined. Error due to such exchange may be largely eliminated by arranging to have t' as much below room temperature as t'' is above room temperature.

The time required for the substance tested to reach thermal equilibrium with the water should be short. This is accomplished

by having the substance in small pieces, and by vigorous stirring.

Apparatus. — The calorimeter consists of two vessels of polished metal placed one within the other and separated by pieces of cork. This device reduces the exchange of heat by conduction and radiation to a small value. The cover of the inner vessel is provided with two holes — one for a sensitive thermometer and one for the handle of the stirrer. The pieces of metal to be tested may serve as a stirrer, being moved up and down by a thread. A steam jacket is used for heating the pieces of metal. By placing the metal in the inner compartment of this jacket while water is kept boiling in the outer compartment, the metal may be heated to the required temperature without being wetted. A thermometer with its bulb inserted into the inner compartment gives the temperature of the metal.

Work to be done. — Weigh out about 200 grams of the metal to be tested, and weigh the inner vessel of the calorimeter.

Preliminary Test. — Put into the calorimeter sufficient water to cover the pieces of metal, and weigh. Having heated the metal to about 90° C., transfer the metal to the calorimeter and note the rise in temperature of the water. From these data determine the amount of water which should be used to make $t'' - t'$ equal, say, to ten degrees.

Final Test. — Put the metal into the steam jacket to heat. Weigh out and put into the calorimeter the required amount of water, and bring it to a temperature five degrees below room temperature. Read the temperature t' of the calorimeter, read the temperature t of the metal in the steam jacket, and quickly transfer the metal to the calorimeter. Keep up a vigorous stirring, and read the highest temperature indicated by the thermometer.

Repeat this final test five or six times.

Ask the instructor as to the metal of the calorimeter and stirrer in order to be able to compute their water equivalents. The water equivalent of the thermometer may be taken as 0.46 times the volume of the immersed portion in cubic centimeters.

Note. — The temperatures cannot be read closer than to about 0.1 degree, so that the weighings need not be made closer than, say, to the nearest tenth-gram.

Computations and results. — Find the specific heat of the metal from each set of observations, find the average of these values, and determine the probable error of this result.

EXPERIMENT 56.

HEAT OF COMBUSTION.

The object of this experiment is to determine the heat of combustion of ordinary illuminating gas or of acetylene.

Apparatus. — The determination of the heat of combustion of coal is a very important practical test, but it is not suited to the elementary laboratory.* The heat of combustion of ordinary illuminating gas, however, may be determined quite accurately with very simple apparatus.

A measured volume of the gas is burned under a water calorimeter, and the products of combustion are drawn through a coil of pipe *cc* by an aspirator as shown in Fig. 72.

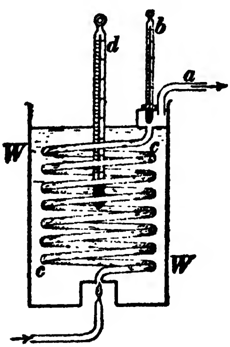


Fig. 72.

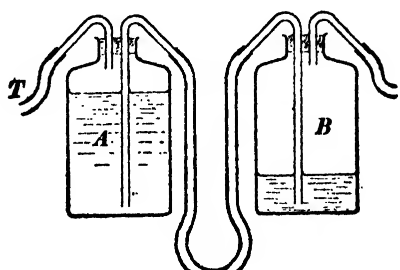


Fig. 73.

The aspirator may be made from two large bottles as shown in Fig. 73; bottle *B* being lowered, the water flows from *A* to

* See Carpenter's "Experimental Engineering" (Wiley & Sons) for a description of the standard practical methods for testing coal.

B, and the tube *T* may be connected to *a*, Fig. 72. The volume of air and burned gas drawn through the pipe *cc*, Fig. 72, may be determined by weighing or measuring the water which flows from *A* to *B*.

The gas to be burned may be stored and measured by means of a gasometer consisting of two bottles arranged like Fig. 73. The gas to be burned is to be drawn into the bottle *B* by lowering *A*, and then driven out of *B* at any desired rate by raising *A*. The rate of flow should be controlled by a pinch-cock.

The temperature of the air should be indicated by a thermometer, the temperature of the outflowing products of combustion should be indicated by the thermometer *b*, Fig. 72, and the temperature of the water in the calorimeter should be indicated by a very accurate thermometer *d*, Fig. 72.

Great care should be exercised to draw a sufficient amount of air through the apparatus to cause complete combustion of the gas. Thus, for the complete combustion of one volume of acetylene, 12½ volumes of air are required and a considerably larger proportion of air than this must be used to insure complete combustion. For example, to burn 50 cubic inches of acetylene gas at least 625 cubic inches of air are required.

Work to be done. — Weigh the calorimeter vessel *WW*, Fig. 72, together with the coiled tube and determine its water equivalent. Then weigh the calorimeter nearly filled with water.

Arrange the gas reservoir and the aspirator ready for operation, observe the reading of thermometer *d* with great accuracy, start the gas burning, read the air temperature and the temperature of the exhaust gases at intervals of one minute during the run, and observe the amount of gas burned, and the volume of exhaust gases drawn into the aspirator. When the water calorimeter has been raised to a temperature as much above that of the air as it was below the air temperature at the start, turn off the gas, and read the thermometer *d* with great care.

It may be necessary to make a preliminary trial in order to

find out how rapidly the air must be drawn through the tube cc in order to insure complete combustion of the gas.

Computations and results. — *If* the exhaust gas were at the same temperature as the air in the above observations, then the amount of heat represented by the observed rise of temperature of the calorimeter would be the heat of combustion of the given amount of gas. A correction for the heat carried off in the exhaust gas may be made as follows: The specific heat of the exhaust gas may be taken to be the same as that of the air, volume for volume, inasmuch as these exhaust gases are mostly nitrogen. The specific heat of air is 0.237 calorie per gram at constant pressure. Therefore, estimating the volume V of the exhaust gas at 0° C., and considering its density to be the same as air, namely, 0.0012 gram per cubic centimeter, we may estimate the heat carried away by the exhaust gas as the product $0.237 \times V \times 0.0012 \times (t' - t'')$, where t' is the mean temperature of the exhaust gas during the progress of the run and t'' is the mean temperature of the air during the run.

Reduce the heat of combustion of the gas to British thermal units per cubic foot.

EXPERIMENT 57.

Latent Heat of Fusion of Ice.

The object of this experiment is to determine the latent heat of fusion of ice.

Theory. — The amount of heat h required to change one gram of ice at 0° C. to water at the same temperature is called the latent heat of fusion of the ice.

Let I grams of dry ice at 0° C. be placed in W grams of warm water at temperature t. The warm water gives up heat enough to melt the ice and to warm the resulting water from 0° C. to the final temperature t' of the mixture. Therefore

$$W(t - t') = Ih + It' \qquad \text{(i)}$$

in which h is the latent heat of fusion of the ice. This equation enables the calculation of h when I and W are known and when t and t' have been observed.

The weight W of water should include the water equivalents of the calorimeter vessel, stirrer, and thermometer; and the initial temperature t of the water should be as much above room temperature as the final temperature t' is below room temperature.

Work to be done. — Weigh the inner vessel of the calorimeter and weigh the stirrer. Fill the vessel half full of water and weigh again. Warm this water to about 15° C. above the temperature of the room. Put the calorimeter vessel into its jacket, observe the temperature carefully, and immediately stir in dry ice until the temperature has fallen to about 15° below room temperature. For the final temperature observe the lowest reading reached by the thermometer, and then weigh the calorimeter vessel so as to determine the amount of ice that has been added.

These observations should be repeated five or six times so as to permit of the determination of the probable error of the result.

Ask the instructor as to the material of the calorimeter vessel and stirrer.

Computations and results. — Calculate the latent heat of fusion h of ice from each set of data, find the mean of these calculated values of h and find the probable error of this result.

EXPERIMENT 58.

Ratio of Specific Heats of Air by Clement and Desormes' Method.

The object of this experiment is to determine the ratio of the specific heat of air at constant pressure to its specific heat at constant volume.

Theory. — The method of Clement and Desormes depends upon a knowledge of the behavior of a gas.

When the temperature of dry air remains unchanged the pressure is inversely proportional to the volume, or the product of

pressure and volume is constant. If, however, the temperature changes, this product changes proportionally. That is,

$$pv = RT \tag{i}$$

in which p is the pressure, v is the volume, T is the absolute temperature of a given amount of air, and R is a constant.

Isothermic Expansion. — A change of v and p with constant T is called *isothermic expansion.* In this case RT is constant, so that the isothermal-process curve is an hyperbola. During isothermic expansion heat must be imparted to the gas to keep it from being cooled by the expansion, and during isothermic compression heat must be abstracted from the gas to keep it from being heated by the compression.

Adiabatic Expansion. — When a gas expands without having heat supplied to it, or when a gas is compressed without having heat abstracted from it, the expansion or compression is called *adiabatic.* Before taking up the discussion of Clement and Desormes' method it is necessary to derive the equation showing the relation between pressure, volume and temperature of a gas during adiabatic expansion or compression. For this purpose it is necessary to consider the two specific heats of the gas, namely the specific heat at constant volume C_v, and the specific heat at constant pressure C_p.

The specific heat at constant volume is the number of heat units required to increase the temperature of one gram of the gas one degree Centigrade without change of volume.

The specific heat at constant pressure is the number of heat units required to increase the temperature of one gram of the gas one degree Centigrade, the gas being so increased in volume that the pressure remains unchanged.

According to Thomson and Joule's experiment, the expanding of a gas cools it only because of the external work done by the gas; that is, to speak in terms of the molecular theory, the particles of the gas have no perceptible attraction for each other. Such an attraction would decrease their velocities slightly as they move

apart, and thereby reduce the temperature of the gas. Therefore any drop in temperature ΔT of one gram of gas with or without expansion means that the gas itself has lost an amount of heat equal to $C_v \cdot \Delta T$, and if the gas has dropped in temperature simply because of the external work, $p \cdot \Delta v$, done in expanding by the amount. Δv, no heat as such being supplied to or abstracted from the gas (adiabatic expansion), then

$$C_v \cdot \Delta T = -p \cdot \Delta v \qquad \text{(ii)}$$

Before reducing this equation, it is desirable to derive a relation between C_v, C_p, and the constant R in equation (i). Suppose a gas to have increased its temperature by the amount ΔT at constant pressure. The heat supplied to the gas is $C_p \cdot \Delta T$, and the portion of this heat which is actually given to the gas is $C_v \cdot \Delta T$, the remainder being transformed into the external work, $p \cdot \Delta v$, which the gas has done by expanding by the amount Δv as its temperature was increased under the constant pressure. Therefore

$$C_p \cdot \Delta T = C_v \cdot \Delta T + p \cdot \Delta v \qquad \text{(iii)}$$

But, since Δv is an increase of volume due to the increase of temperature ΔT at constant pressure, we have from equation (i)

$$v = \frac{RT}{p}$$

or

$$\Delta v = \frac{R}{p} \cdot \Delta T$$

or

$$p \cdot \Delta v = R \cdot \Delta T$$

Therefore equation (iii) becomes, after canceling ΔT

$$C_p - C_v = R \qquad \text{(iv)}$$

Returning now to equation (ii), we may substitute from (i) the value of p, and after reduction, we have:

$$\frac{\Delta T}{T} = -\frac{R}{C_v} \cdot \frac{\Delta v}{v}$$

or, using the value of R from (iv),

$$\frac{\Delta T}{T} = -\frac{C_p - C_v}{C_v} \cdot \frac{\Delta v}{v} \tag{v}$$

Let

$$k = \frac{C_p}{C_v} \tag{vi}$$

then

$$\frac{C_p - C_v}{C} = k - 1$$

and equation (v) becomes

$$\frac{\Delta T}{T} = -(k-1)\frac{\Delta v}{v}$$

or

$$\frac{\Delta T}{T} + (k-1)\frac{\Delta v}{v} = 0$$

or

$$\log T + (k-1)\log v = \text{constant}$$

or

$$Tv^{k-1} = \text{constant} \tag{vii}$$

which expresses the relation between temperature and volume of a gas during adiabatic expansion.

Substituting in (vii) the value of v from (i), remembering that R is a constant, and reducing to terms of the first power of T, we have

$$Tp^{\frac{1-k}{k}} = \text{constant} \tag{viii}$$

which expresses the relation between temperature and pressure of a gas during adiabatic expansion.

Clement and Desormes' method for the experimental determination of k.—A vessel filled with the gas at pressure p_1 and temperature T_1 is opened to the air allowing the pressure to drop quickly to atmospheric pressure p_2, the temperature falling at the same time to T_2. Equation (viii) gives

$$T_1 p_1^{\frac{1-k}{k}} = T_2 p_2^{\frac{1-k}{k}} \tag{ix}$$

The vessel is then quickly closed and allowed to stand until it reaches a uniform temperature T_3, when the pressure is p_3, so that from Gay Lussac's law

$$\frac{T_2}{p_2} = \frac{T_3}{p_3} \qquad \text{(x)}$$

Substituting the value of T_2 from (x) in (ix), and reducing, we have

$$\left(\frac{p_1}{p_2}\right)^{\frac{1-k}{k}} = \frac{T_3 p_2}{T_1 p_3} \qquad \text{(xi)}$$

from which k may be calculated when p_1, p_2, p_3, T_1 and T have been observed.

In practice the difference between T_1 and T_3 is usually small, both being near the temperature of the room. By allowing time for the entrapped air to settle to room temperature we may make $T_1 = T_3$. Equation (xi) then becomes

$$\left(\frac{p_1}{p_2}\right)^{\frac{1-k}{k}} = \frac{p^2}{p_3} \qquad \text{(xii)}$$

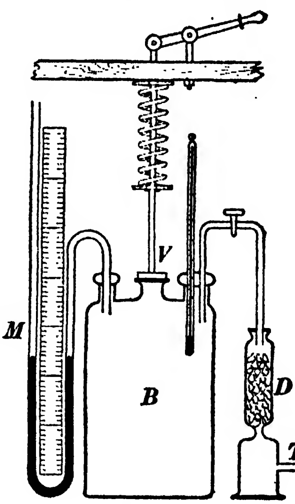


Fig. 74.

Apparatus. — Air is pumped through a dryer D into an air-tight reservoir B, Fig. 74. A stop-cock is provided to prevent leakage back through the drier and pump after the desired pressure has been obtained in the reservoir. At the top of the reservoir is a wide valve V by opening which the pressure in the reservoir is suddenly relieved. The length of time this valve is open must be neither too short nor too long; if too short, the pressure in the reservoir will not have time to fall to atmospheric pressure; if too

long, there is time for change of temperature and consequent further expansion of the air in the reservoir, thus making the value of p_3 (above) less than it should be.

Work to be done. — Close the valve on the containing vessel and pump air very slowly and carefully * through the drier until the pressure in the vessel is as large as can be conveniently indicated by the attached manometer. Then close the stopcock and allow the apparatus to stand until the containing vessel is at a uniform temperature.

Read the upper and lower levels of the mercury in the manometer tube. Then open and close the valve, making sure that it is closed tight. After the reservoir has again settled to room temperature, again read the manometer.

Repeat this operation five or six times for each of the following cases: (1) allowing, say, one second to elapse between opening and closing the valve, (2) allowing about one quarter second to elapse, and (3) allowing as little time as possible.

The reading of the barometer should be taken at the beginning and at the end of the exercise.

Computations and results. — The values of p_1, p_2, and p_3 in equations (xi) and (xii) are to be obtained by adding the reading b of the barometer to the observed differences of pressure as indicated by the manometer M, Fig. 74. Calculate the value of the ratio k for each of the three cases specified above, and compare the results.

EXPERIMENT 59.

Determination of Vapor Pressures.

When a liquid is placed in a closed vessel it vaporizes until the pressure of its vapor reaches the maximum pressure for the given temperature. The object of this experiment is to determine this maximum pressure of the vapor of a given liquid at various temperatures.

* Do not blow the mercury out of the manometer tube.

Apparatus. — The apparatus consists of a special barometer in which the space above the mercury column is filled with a liquid and its vapor, as shown in Fig. 75. The upper end of the special barometer is enclosed in a water jacket into which a thermometer is inserted. By passing warm water through the jacket the temperature of the liquid and vapor may be adjusted as desired, and the depression of the mercury column C (due account being taken of the barometer reading at the time) gives the pressure of the vapor.

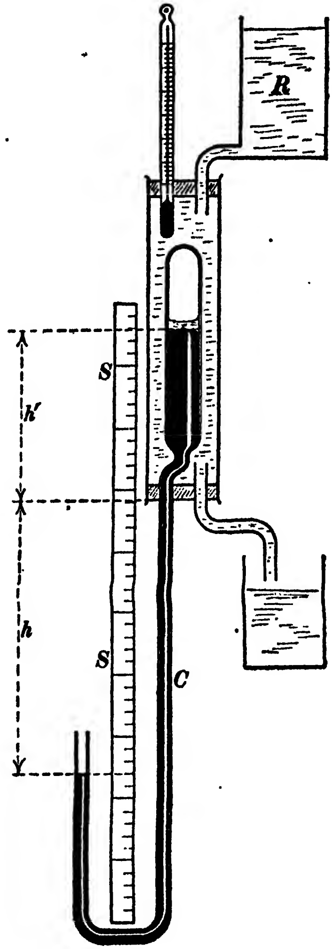


Fig. 75.

Work to be done. — By means of a reading telescope and the scale SS read the depth a of liquid on top of the mercury column, and read the position of the point at which the mercury column emerges from the jacket.

Fill the reservoir R with ice water, and let the water flow through the jacket. After the water has been flowing for a time, read the thermometer, read the positions of the upper and lower levels of the mercury column and again read the thermometer. The readings of the mercury column are to be made with the reading telescope.

Warm the water in the reservoir successively to about 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80° and 90° C., and take readings for each of these temperatures, as above.

The temperature of the room and the barometer reading should be taken at the beginning and at the end of the exercise.

Computations and results. — Determine the values of h and h', Fig. 75, from the readings taken at each temperature. The part h of mercury column is at room temperature, whereas the part h' is at the temperature of the water jacket. Both must be reduced to 0° C. by multiplying by $(1 - 0.000181t)$ where t is the actual temperature of the portion of the mercury column. Find the mercury equivalent of the depth a of the liquid at the top of the mercury column by dividing a by d/D, where d is the density of the liquid and D is the density of mercury.

Let H be the sum of these corrected values of h, h' and a and let b be the barometer reading reduced to zero. Then the required vapor pressure is

$$p = b - H$$

Calculate the values of vapor pressure in this way for each temperature, tabulate the corresponding values of vapor pressure and temperature, and plot a curve of which the abscissas represent temperatures and ordinates represent vapor pressures.

EXPERIMENT 60.

HYGROMETRY. THE USE OF WET AND DRY BULB THERMOMETERS.

The object of this experiment is to afford an exercise in the use of wet and dry bulb thermometers for determining the humidity of the atmosphere.

The *relative humidity* of the air is the amount of moisture it contains expressed in hundredths of what it would contain if saturated at the given temperature.

The *dew point* is the temperature to which the air would have to be cooled in order to become saturated with the moisture which it contains. If the air is cooled below the dew point, part of the moisture condenses.

The *force of vapor* is that part of the pressure of the air which is due to the water vapor that is present.

The *absolute humidity* of the air is usually expressed as the number of grains of water contained in each cubic foot.

Wet and dry bulb thermometers.—A wet bulb thermometer and a dry bulb thermometer indicate the same temperature only when the air is saturated with moisture. When the air is not saturated, the wet bulb thermometer is cooled by evaporation.

For a given state of the air as to temperature and humidity, the readings of wet and dry bulb thermometers are definite, and if relative humidity, dew point, absolute humidity, and force of vapor are once for all determined for various readings of wet and dry bulb thermometers, then the tabular results of these determinations may be used to find the various quantities (relative humidity, dew point, etc.) from the observed readings of wet and dry bulb thermometers. This is the method usually employed in meteorology for determining the degree of humidity of the air.

The wet and dry bulb thermometers should be exposed to a moderate draught of air, and if observations are taken indoors, an artificial draught of air should be produced by means of a fan.

Work to be done.—Observe the readings of a wet and a dry bulb thermometer, and find from tables the relative humidity, dew point, force of vapor, and absolute humidity.*

Observe the dew point directly by stirring ammonium chloride into water contained in a polished metal vessel. The dissolving of the salt lowers the temperature, which is observed by means of a thermometer at the instant that a film of moisture is seen to condense on the vessel. It is advisable to make a preliminary

* See publication entitled "W. B. No. 235" of the United States Department of Agriculture (price, 10 cents). Pages 15 to 56 of this publication give the temperatures of the dew point corresponding to observed readings of wet and dry bulb thermometers, and the saturation value of the vapor pressure *e* corresponding to the actual temperature of the air. Pages 57 to 82 give relative humidities in per cent. corresponding to observed readings of wet and dry bulb thermometers. Pages 83 and 84 give the number of grains of water per cubic foot at different temperatures and different degrees of saturation (relative humidities). In the accurate use of these tables the approximate barometer reading should be taken.

trial to find the approximate value of the dew point, then a second trial in which the temperature of the vessel is lowered very slowly gives a more accurate determination.

EXPERIMENT 61.

FLASH TESTS.

The object of this experiment is to determine what is called the flash point of a sample of kerosene.

Theory. — Imagine a closed vessel at temperature t containing dry air at pressure p, and a small amount of liquid. The liquid will evaporate until the pressure finally becomes $p + p'$, where p' is the maximum pressure that the vapor of the given liquid can exert at temperature t. When this final pressure is reached the air in the vessel is said to be saturated with the vapor of the liquid. The gas in the vessel will be a mixture of dry air and vapor such as would be obtained by mixing p volumes of dry air measured at standard pressure with p' volumes of vapor measured at standard pressure.* As the temperature t rises p' increases and the proportion of vapor in the mixture becomes larger and larger.

When an inflammable gas is mixed with air, the mixture will not explode until the proportion of gas to air has reached a certain amount which varies with the nature of the gas. Thus, one part of acetylene to twenty-five of air is an explosive mixture, while a much larger proportion of gasoline vapor is required to give an explosive mixture. If a vessel contains air and a volatile oil such as kerosene, the air soon becomes saturated with the oil vapor and if the temperature of the vessel is slowly raised the proportion of oil vapor increases until, at a certain temperature, the saturated air becomes explosive. This temperature is called the *flash point* of the oil. An oil, to be safe for use in lamps, should have a high flash point, and the law requires that the flash point of commercial kerosene shall be above a certain minimum.

*Dalton's law.

Apparatus. — The value of the flash point of a sample of oil as determined by test depends upon the rapidity of heating of the vessel in which the mixture of oil and vapor collects, upon the size of the aperture through which the igniting flame is applied, and perhaps on other circumstances. Therefore, commercial tests are carried out with a standard form of apparatus and according to a standard régime. This experiment is intended to be performed with the oil tester as adopted by the State Boards of Health of New York and Iowa. In this tester a small metal vessel with a glass cover is partly filled with the sample of oil to be tested and slowly heated in a water-bath. The thermometer for indicating the temperature of the oil is inserted through a cork which fits in a round hole in the center of the glass cover, and the torch or match for igniting the oil vapor is inserted through a hole at one side of the glass cover. The water-bath is heated by means of a small alcohol lamp, the wick of which is adjusted to give a small flame so that the rate of heating of the water-bath shall be between 2 and 3° per minute. An ignited wooden toothpick is used for the torch for igniting the vapor to denote the flash test.

Work to be done.* — Remove the oil cup and fill the water-bath with cold water up to the rivet mark on the inside. Replace the oil cup and pour in oil to bring the surface of the oil ⅛ of an inch below the point where the cup widens out to form the vapor chamber. Do not allow the oil to wet the sides of the oil cup above the level at which the oil finally stands in the cup. Place the cork in the center hole of the glass cover and insert the thermometer so that the bulb of the thermometer is covered by the oil.

Having previously adjusted the alcohol lamp so as to warm the water-bath at the desired rate, light the lamp and allow the water-bath and oil vessel to be slowly heated. When the temperature of the oil has reached 85° F. insert the torch into the

* The directions here given are taken from the recommendations of the New York State Board of Health.

opening in the glass cover passing it in at such an angle that it may clear the cover and stand about half-way between the surface of the oil and the cover. The torch should be introduced with a fairly rapid but uniform motion and then immediately withdrawn. The torch should be inserted in this way for every 2° F. rise of temperature until the temperature has reached 95° F. Then the lamp should be removed and the testing should be made for each degree of rise of temperature until 100° F. is reached. The reason for proceeding so slowly near 100° F. is that most commercial kerosene has a flash point at about 100° F.

If the oil does not flash at or below 100° F. the lamp should be replaced and the testing repeated for every 2° of continued rise of temperature.

The appearance of a slight bluish flame as seen through the glass cover shows that the flash point has been reached.

In every case, note the temperature of the oil before introducing the torch and avoid bringing the torch into contact with the oil.

The oil from a previous test should be thoroughly wiped out of the cup before a new sample is put in.

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